Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

AREA

In Questions 1–11, choose the alternative that gives the area of the region whose boundaries are given.

1. The curve of $y = x^2$, $y = 0$, $x = -1$, and $x = 2$.

(A) $\frac{11}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 5 (E) none of these

2. The parabola $y = x^2 - 3$ and the line $y = 1$.

(A) $\frac{8}{3}$ (B) 32 (C) $\frac{32}{3}$ (D) $\frac{16}{3}$ (E) none of these

3. The curve of $x = y^2 - 1$ and the y-axis.

(A) $\frac{4}{3}$ (B) $\frac{2}{3}$ (C) $\frac{8}{3}$ (D) $\frac{1}{2}$ (E) none of these

4. The parabola $y^2 = x$ and the line $x + y = 2$.

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{11}{6}$ (D) $\frac{9}{2}$ (E) $\frac{29}{6}$

5. The curve of $y = \frac{4}{x^2 + 4}$, the x-axis, and the vertical lines $x = -2$ and $x = 2$.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) 2π (D) π (E) none of these

6. The parabolas $x = y^2 - 5y$ and $x = 3y - y^2$.

(A) $\frac{32}{3}$ (B) $\frac{139}{6}$ (C) $\frac{64}{3}$ (D) $\frac{128}{3}$ (E) none of these

7. The curve of $y = \frac{2}{x}$ and $x + y = 3$.

(A) $\frac{1}{2} - 2\ln 2$ (B) $\frac{3}{2}$ (C) $\frac{1}{2} - \ln 4$

(D) $\frac{5}{2}$ (E) $\frac{3}{2} - \ln 4$

8. In the first quadrant, bounded below by the x-axis and above by the curves of $y = \sin x$ and $y = \cos x$.

(A) $2 - \sqrt{2}$ (B) $2 + \sqrt{2}$ (C) 2 (D) $\sqrt{2}$ (E) $2\sqrt{2}$

9. Bounded above by the curve $y = \sin x$ and below by $y = \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{5\pi}{4}$

(A) $2\sqrt{2}$ **(B)** $\frac{2}{\sqrt{2}}$ **(C)** $\frac{1}{2\sqrt{2}}$

(D) $2(\sqrt{2}-1)$ **(E)** $2(\sqrt{2}+1)$

10. The curve $y = \cot x$, the line $x = \frac{\pi}{4}$, and the x-axis.

(A) $\ln 2$ **(B)** $\frac{1}{2}\ln\frac{1}{2}$ **(C)** 1 **(D)** $\frac{1}{2}\ln 2$ **(E)** 2

11. The curve of $y = x^3 - 2x^2 - 3x$ and the x-axis.

(A) $\frac{28}{3}$ **(B)** $\frac{79}{6}$ **(C)** $\frac{45}{4}$ **(D)** $\frac{71}{6}$ **(E)** none of these

12. The total area bounded by the cubic $x = y^3 - y$ and the line $x = 3y$ is equal to

(A) 4 **(B)** $\frac{16}{3}$ **(C)** 8 **(D)** $\frac{32}{3}$ **(E)** 16

13. The area bounded by $y = e^x$, $y = 2$, and the y-axis is equal to

(A) $3 - e$ **(B)** $e^2 - 1$ **(C)** $e^2 + 1$

(D) $2\ln 2 - 1$ **(E)** $2\ln 2 - 3$

14. The area enclosed by the ellipse with parametric equations $x = 2\cos\theta$ and $y = 3\sin\theta$ equals

(A) 6π **(B)** $\frac{9}{2}\pi$ **(C)** 3π **(D)** $\frac{3}{2}\pi$ **(E)** none of these

15. The area enclosed by one arch of the cycloid with parametric equations $x = \theta - \sin\theta$ and $y = 1 - \cos\theta$ equals

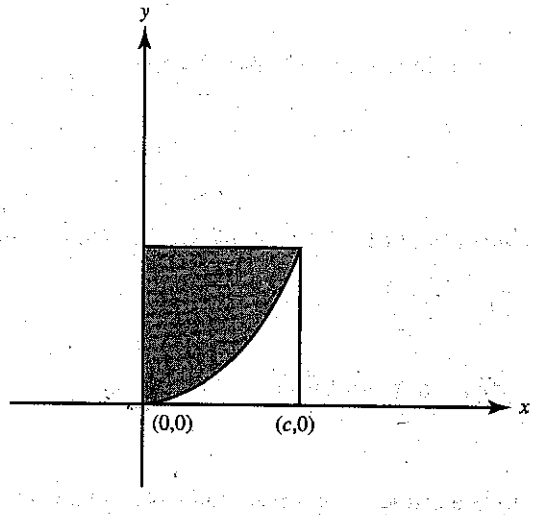
(A) $\frac{3\pi}{2}$ **(B)** 3π **(C)** 2π **(D)** 6π **(E)** none of these

16. The area enclosed by the curve $y^2 = x(1-x)$ is given by

(A) $2\int_0^1 x\sqrt{1-x}\,dx$ **(B)** $2\int_0^1 \sqrt{x-x^2}\,dx$ **(C)** $4\int_0^1 \sqrt{x-x^2}\,dx$

(D) π **(E)** 2π

17. The figure below shows part of the curve of $y = x^3$ and a rectangle with two vertices at (0, 0) and $(c, 0)$. What is the ratio of the area of the rectangle to the shaded part of it above the cubic?



(A) 3:4 **(B)** 5:4 **(C)** 4:3 **(D)** 3:1 **(E)** 2:1

VOLUME

In Questions 18–24 the region whose boundaries are given is rotated about the line indicated. Choose the alternative that gives the volume of the solid generated.

18. $y = x^2$, $x = 2$, and $y = 0$; about the x-axis.

(A) $\frac{64\pi}{3}$ **(B)** 8π **(C)** $\frac{8\pi}{3}$ **(D)** $\frac{128\pi}{5}$ **(E)** $\frac{32\pi}{5}$

19. $y = x^2$, $x = 2$, and $y = 0$; about the y-axis.

(A) $\frac{16\pi}{3}$ **(B)** 4π **(C)** $\frac{32\pi}{5}$ **(D)** 8π **(E)** $\frac{8\pi}{3}$

20. The first quadrant region bounded by $y = x^2$, the y-axis, and $y = 4$; about the y-axis.

(A) 8π **(B)** 4π **(C)** $\frac{64\pi}{3}$ **(D)** $\frac{32\pi}{3}$ **(E)** $\frac{16\pi}{3}$

21. $y = x^2$ and $y = 4$; about the x-axis.

(A) $\frac{64\pi}{5}$ **(B)** $\frac{512\pi}{15}$ **(C)** $\frac{256\pi}{5}$

(D) $\frac{128\pi}{5}$ **(E)** none of these

22. $y = x^2$ and $y = 4$; about the line $y = 4$.

(A) $\frac{256\pi}{15}$ (B) $\frac{256\pi}{5}$ (C) $\frac{512\pi}{5}$ (D) $\frac{512\pi}{15}$ (E) $\frac{64\pi}{3}$

23. An arch of $y = \sin x$ and the x-axis; about the x-axis.

(A) $\frac{\pi}{2}\left(\pi - \frac{1}{2}\right)$ (B) $\frac{\pi^2}{2}$ (C) $\frac{\pi^2}{4}$ (D) π^2 (E) $\pi(\pi - 1)$

24. A trapezoid with vertices at (2, 0), (2, 2), (4, 0), and (4, 4); about the x-axis.

(A) $\frac{56\pi}{3}$ (B) $\frac{128\pi}{3}$ (C) $\frac{92\pi}{3}$

(D) $\frac{112\pi}{3}$ (E) none of these

25. The base of a solid is a circle of radius a, and every plane section perpendicular to a diameter is a square. The solid has volume

(A) $\frac{8}{3}a^3$ (B) $2\pi a^3$ (C) $4\pi a^3$ (D) $\frac{16}{3}a^3$ (E) $\frac{8\pi}{3}a^3$

26. The base of a solid is the region bounded by the parabola $x^2 = 8y$ and the line $y = 4$, and each plane section perpendicular to the y-axis is an equilateral triangle. The volume of the solid is

(A) $\frac{64\sqrt{3}}{3}$ (B) $64\sqrt{3}$ (C) $32\sqrt{3}$

(D) 32 (E) none of these

27. The base of a solid is the region bounded by $y = e^{-x}$, the x-axis, the y-axis, and the line $x = 1$. Each cross section perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{e^2}{2}$ (B) $e^2 - 1$ (C) $1 - \frac{1}{e^2}$

(D) $\frac{e^2 - 1}{2}$ (E) $\frac{1}{2}\left(1 - \frac{1}{e^2}\right)$

BC ONLY

ARC LENGTH

28. The length of the arc of the curve $y^2 = x^3$ cut off by the line $x = 4$ is

(A) $\frac{4}{3}(10\sqrt{10} - 1)$ (B) $\frac{8}{27}(10^{3/2} - 1)$ (C) $\frac{16}{27}(10^{3/2} - 1)$

(D) $\frac{16}{27}10\sqrt{10}$ (E) none of these