

AP Calculus
IVT, Continuity Packet

Name _____

Date _____

1. Find the interval over which $2^x = \sin(2x) + 5$ must have a solution by applying the Intermediate Value Theorem.

- a) $[-2, -1]$ b) $[-1, 0]$ c) $[0, 1]$
d) $[1, 2]$ e) $[2, 3]$

2. Given a function defined by $f(x) = \frac{3x - 12}{x^2 - 6x + 8}$, for what value(s) of x is the function discontinuous?

- a) 4 only b) 2 c) 2, 4
d) -4 only e) -4, -2

3. Which of the following functions are continuous for all real numbers x ?

I. $y = -\frac{2}{x-3}$

II. $y = e^x$

III. $y = \csc x$

- a) I and III only b) II only
c) II and III only d) I and II only
e) I only

4. Which of the following functions are continuous for all real numbers x ?

I. $y = \frac{x^2 - 13}{x^2 + 7}$

II. $y = -\frac{5}{x^2}$

III. $y = |3x + 11|$

- a) I and III only b) I only
c) III only d) I and II only
e) II and III only

5. Which of the following functions are continuous for all real numbers x ?

I. $y = (x + 2)^2$

II. $y = \sqrt{2x^2 - x^3}$

III. $y = 4 \ln x$

- a) I only b) II only
c) I and II only d) I and III only
e) I, II, and III

AP Calculus
IVT, Continuity Packet

Name _____

Date _____

6. Which of the following functions are continuous for all real numbers x ?

I. $y = 3(x - 2)^2$

II. $y = \sqrt{x^2 - 2x^3}$

III. $y = 4 \ln 5x$

- a) I only b) II only
c) I and II only d) I and III only
e) I, II, and III

7. Which of the following functions are continuous for all real numbers x ?

I. $y = \frac{1}{5 + 3x^4}$

II. $y = 3x \cos x$

III. $y = \frac{4}{e^{x^2/3}}$

- a) III only b) II and III only
c) I and III only d) I and II only
e) I, II, and III

8. For what value(s) of x does the function defined by $f(x) = \frac{x^2 + 4x - 21}{x^2 + 10x + 21}$ have a removable discontinuity?

- a) -7 only b) -3 only c) 7 only
d) -7 and -3 e) 7 and 3

9. Let f be defined as follows:

$$f(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & \text{for } x \neq 4, \\ 15 & \text{for } x = 4 \end{cases}$$

Which of the following are true about f ?

- I. $\lim_{x \rightarrow 4} f(x)$ exists
II. $f(4)$ exists
III. $f(x)$ is continuous at $x = 4$

- a) None b) I only
c) II only d) I and II only
e) I, II, and III

AP Calculus
IVT, Continuity Packet

Name _____

Date _____

10. Consider f as defined below:

$$f(x) = \begin{cases} \frac{x^2 - 144}{x - 12} & \text{for } x \neq 12, \\ 24 & \text{for } x = 12 \end{cases}$$

Which of the following are true about f ?

- I. $\lim_{x \rightarrow 12} f(x)$ exists
- II. $f(12)$ exists
- III. $f(x)$ is continuous at $x = 12$

- a) None
- b) I only
- c) II only
- d) I and II only
- e) I, II, and III

11. Let f be defined as follows:

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{for } x \neq a, \\ 2a & \text{for } x = a \end{cases}$$

Which of the following are true about f ?

- I. $\lim_{x \rightarrow a} f(x)$ exists
- II. $f(a)$ exists
- III. $f(x)$ is continuous at $x = a$

- a) None
- b) I only
- c) II only
- d) I and II only
- e) I, II, and III

12. Let f be defined as follows:

$$f(x) = \begin{cases} x^2 + 5 & \text{for } x > 5, \\ 3ax & \text{for } x \leq 5 \end{cases}$$

For what value of a is the function continuous?

- a) 2
- b) 10
- c) 5
- d) 15
- e) 30

13. $f(x) = \begin{cases} x^2 + 8 & \text{for } x < 8, \\ a^2x & \text{for } x \geq 8 \end{cases}$

For what value(s) of a is the function continuous?

- a) ± 3
- b) 9
- c) 64
- d) -9
- e) 18

14. Let f be a continuous function on $[2, 4]$ and have the values shown.

The equation $f(x) = k$ must have at least 2 solutions on $[2, 4]$ for which value(s) of k ?

a) $k > 9$

b) 7

c) $0 < k < 5$

d) $k > 5$

e) $5 < k < 9$

| | | | |
|--------|---|---|---|
| x | 2 | 3 | 4 |
| $f(x)$ | 5 | 0 | 9 |

AP Calculus
IVT, Continuity Packet

Name _____

Date _____

15. Let f be a continuous function on $[1, 5]$ and have the values shown.

The equation $f(x) = k$ must have at least 2 solutions on $[1, 5]$ for which value(s) of k ?

- a) $k > 8$
 b) 5
 c) $1 < k < 4$
 d) $k > 4$
 e) $4 < k < 8$

| | | | |
|--------|---|---|---|
| x | 1 | 3 | 5 |
| $f(x)$ | 4 | 1 | 8 |

16. Consider $f(x) = \begin{cases} x^2 - 5 & \text{for } x < 0, \\ 3 & \text{for } x = 0, \\ x^2 + 5 & \text{for } x > 0 \end{cases}$

- a) $\lim_{x \rightarrow 0^+} f(x) = \underline{\hspace{2cm}}$
 b) $\lim_{x \rightarrow 0^-} f(x) = \underline{\hspace{2cm}}$
 c) $\lim_{x \rightarrow 3} f(x) = \underline{\hspace{2cm}}$
 d) Where is $f(x)$ discontinuous?
 e) If a function is continuous at $x = a$, does this necessarily mean that $\lim_{x \rightarrow a}$ exists?

17. Consider $f(x) = \begin{cases} x + c & \text{for } x < -2, \\ cx^2 + 7 & \text{for } x \geq -2 \end{cases}$

For what value of the constant c is f continuous for all real numbers?

18. $\lim_{x \rightarrow 0^-} \frac{3}{x}$ is

- a) 1 b) 2 c) $-\infty$ d) 0 e) ∞

19. $\lim_{x \rightarrow 0^+} \frac{5}{x^3}$ is

- a) 1 b) $-\infty$ c) 0 d) e e) ∞

20. $\lim_{x \rightarrow 7^-} \frac{x + 9}{x - 7}$ is

- a) 1 b) 2 c) $-\infty$ d) 0 e) ∞

1.
Answer: e
CodePath: EAS.APC.C.G.3
2.
Answer: c
CodePath: EAS.APC.C.F.1
3.
Answer: b
CodePath: EAS.APC.C.F.6
4.
Answer: a
CodePath: EAS.APC.C.F.8
5.
Answer: a
CodePath: EAS.APC.C.F.9
6.
Answer: a
CodePath: EAS.APC.C.F.10
7.
Answer: e
CodePath: EAS.APC.C.F.12
8.
Answer: a
CodePath: EAS.APC.C.F.16
9.
Answer: d
CodePath: EAS.APC.C.F.18
10.
Answer: e
CodePath: EAS.APC.C.F.24
11.
Answer: e
CodePath: EAS.APC.C.F.28
12.
Answer: a
CodePath: EAS.APC.C.F.31
13.
Answer: a
CodePath: EAS.APC.C.F.33
14.
Answer: c
CodePath: EAS.APC.C.F.37

15.
Answer: c
CodePath: EAS.APC.C.F.38
16.
Answer: 5, -5, 4, at 0, yes
CodePath: EAS.APC.C.F.44
17.
Answer: -9
CodePath: EAS.APC.C.F.46
18.
Answer: c
CodePath: EAS.APC.C.D.1
19.
Answer: e
CodePath: EAS.APC.C.D.6
20.
Answer: c
CodePath: EAS.APC.C.D.12