

**AP[®] CALCULUS AB
2003 SCORING GUIDELINES**

Question 2

A particle moves along the x -axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

- (a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?
- (b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.
- (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.
- (d) During the time interval $0 \leq t \leq 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

- (a) $a(2) = v'(2) = 1.587$ or 1.588
 $v(2) = -3\sin(2) < 0$
 Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.

- 2 : { 1 : $a(2)$
 1 : speed decreasing
 with reason

- (b) $v(t) = 0$ when $\frac{t^2}{2} = \pi$
 $t = \sqrt{2\pi}$ or 2.506 or 2.507
 Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.

- 2 : { 1 : $t = \sqrt{2\pi}$ only
 1 : justification

- (c) Distance = $\int_0^3 |v(t)| dt = 4.333$ or 4.334

- 3 : { 1 : limits
 1 : integrand
 1 : answer

- (d) $\int_0^{\sqrt{2\pi}} v(t) dt = -3.265$
 $x(\sqrt{2\pi}) = x(0) + \int_0^{\sqrt{2\pi}} v(t) dt = -2.265$
 Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265.

- 2 : { 1 : \pm (distance particle travels
 while velocity is negative)
 1 : answer

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Question 3

A particle moves along the y -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$.

At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

- (a) Find the acceleration of the particle at time $t = 2$.
- (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
- (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.
- (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

(a) $a(2) = v'(2) = -0.132$ or -0.133

1 : answer

(b) $v(2) = -0.436$

Speed is increasing since $a(2) < 0$ and $v(2) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $\tan^{-1}(e^t) = 1$

$t = \ln(\tan(1)) = 0.443$ is the only critical value for y .

$v(t) > 0$ for $0 < t < \ln(\tan(1))$

$v(t) < 0$ for $t > \ln(\tan(1))$

$y(t)$ has an absolute maximum at $t = 0.443$.

3 : $\left\{ \begin{array}{l} 1 : \text{sets } v(t) = 0 \\ 1 : \text{identifies } t = 0.443 \text{ as a candidate} \\ 1 : \text{justifies absolute maximum} \end{array} \right.$

(d) $y(2) = -1 + \int_0^2 v(t) dt = -1.360$ or -1.361

The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

4 : $\left\{ \begin{array}{l} 1 : \int_0^2 v(t) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{value of } y(2) \\ 1 : \text{answer with reason} \end{array} \right.$

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Question 6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

- (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.
- (b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

(a) $g'(x) = ae^{ax} + f'(x)$
 $g'(0) = a - 4$

$$g''(x) = a^2 e^{ax} + f''(x)$$

$$g''(0) = a^2 + 3$$

$$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$$

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx)f(x)$
 $h'(0) = f'(0)\cos(0) - k\sin(0)f(0) = f'(0) = -4$
 $h(0) = \cos(0)f(0) = 2$
The equation of the tangent line is $y = -4x + 2$.

$$5 : \begin{cases} 2 : h'(x) \\ 1 : h'(0) \\ 3 : \begin{cases} 1 : h(0) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$

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Question 4

A particle moves along the x -axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$.

- (a) Find the time t at which the particle is farthest to the left. Justify your answer.
 (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

- (a) $x'(t) = -e^{-t} \sin t + e^{-t} \cos t = e^{-t} (\cos t - \sin t)$
 $x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on
 $0 \leq t \leq 2\pi$ for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.

The candidates for the absolute minimum are at
 $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$, and 2π .

t	$x(t)$
0	$e^0 \sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}} \sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}} \sin\left(\frac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi} \sin(2\pi) = 0$

The particle is farthest to the left when $t = \frac{5\pi}{4}$.

- (b) $x''(t) = -e^{-t} (\cos t - \sin t) + e^{-t} (-\sin t - \cos t)$
 $= -2e^{-t} \cos t$

$$\begin{aligned} Ax''(t) + x'(t) + x(t) &= A(-2e^{-t} \cos t) + e^{-t} (\cos t - \sin t) + e^{-t} \sin t \\ &= (-2A + 1)e^{-t} \cos t \\ &= 0 \end{aligned}$$

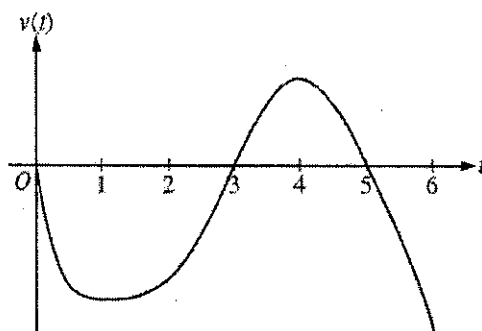
Therefore, $A = \frac{1}{2}$.

$$5: \begin{cases} 2: x'(t) \\ 1: \text{sets } x'(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$$

$$4: \begin{cases} 2: x''(t) \\ 1: \text{substitutes } x''(t), x'(t), \text{ and } x(t) \\ \quad \text{into } Ax''(t) + x'(t) + x(t) \\ 1: \text{answer} \end{cases}$$

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Question 4



Graph of v

A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

- (a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- (b) For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
- (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
- (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

- (a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$.

$$x(3) = -2 + \int_0^3 v(t) dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t) dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.

- (b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$.

By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.

- (c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.
- (d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.

$$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t) dt \\ 1 : \text{conclusion} \end{cases}$$

$$3 : \begin{cases} 1 : \text{positions at } t = 3, t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$$

1 : answer with reason

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

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Question 1

For $0 \leq t \leq 6$, a particle is moving along the x -axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin(e^{t/4}) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos(e^{t/4})$ and $x(0) = 2$.

- (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.
 (b) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.
 (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.
 (d) For $0 \leq t \leq 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337$, $a(5.5) = -1.35851$

The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.

2 : conclusion with reason

(b) Average velocity $= \frac{1}{6} \int_0^6 v(t) dt = 1.949$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Distance $= \int_0^6 |v(t)| dt = 12.573$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$.
 $v(t)$ changes sign from positive to negative at time $t = b$.
 $x(b) = 2 + \int_0^b v(t) dt = 14.134$ or 14.135

3 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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Question 6

For $0 \leq t \leq 12$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = \cos\left(\frac{\pi}{6}t\right). \text{ The particle is at position } x = -2 \text{ at time } t = 0.$$

- (a) For $0 \leq t \leq 12$, when is the particle moving to the left?
 (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.
 (c) Find the acceleration of the particle at time t . Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.
 (d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3, 9$

The particle is moving to the left when $v(t) < 0$.
 This occurs when $3 < t < 9$.

(b) $\int_0^6 |v(t)| dt$

(c) $a(t) = -\frac{\pi}{6} \sin\left(\frac{\pi}{6}t\right)$

$$a(4) = -\frac{\pi}{6} \sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$$

$$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt$
 $= -2 + \left[\frac{6}{\pi} \sin\left(\frac{\pi}{6}t\right)\right]_0^4$
 $= -2 + \frac{6}{\pi} \left[\sin\left(\frac{2\pi}{3}\right) - 0\right]$
 $= -2 + \frac{6}{\pi} \cdot \frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$

2 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

1 : answer

3 : $\begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

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Question 2

A particle moves along a straight line. For $0 \leq t \leq 5$, the velocity of the particle is given by

$v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

- (a) Find all values of t in the interval $2 \leq t \leq 4$ for which the speed of the particle is 2.
- (b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.
- (c) Find all times t in the interval $0 \leq t \leq 5$ at which the particle changes direction. Justify your answer.
- (d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

- (a) Solve $|v(t)| = 2$ on $2 \leq t \leq 4$.
 $t = 3.128$ (or 3.127) and $t = 3.473$

2 : $\begin{cases} 1 : \text{considers } |v(t)| = 2 \\ 1 : \text{answer} \end{cases}$

(b) $s(t) = 10 + \int_0^t v(x) dx$

$$s(5) = 10 + \int_0^5 v(x) dx = -9.207$$

2 : $\begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$

- (c) $v(t) = 0$ when $t = 0.536033, 3.317756$
 $v(t)$ changes sign from negative to positive at time $t = 0.536033$.
 $v(t)$ changes sign from positive to negative at time $t = 3.317756$.

Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

3 : $\begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$

- (d) $v(4) = -11.475758 < 0$, $a(4) = v'(4) = -22.295714 < 0$

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

2 : conclusion with reason