2014-2015
AB CALC
HOPKINS
PRETEST
DERIVATIVES

A.P. Calculus ~~Test Two~~
Section One
Multiple-Choice
Calculators Allowed
Time—45 minutes
Number of Questions—15

The scoring for this section is determined by the formula

$$[C - (0.25 \times I)] \times 1.8$$

where C is the number of correct responses and I is the number of incorrect responses. An unanswered question earns zero points. The maximum possible points earned on this section is 27, which represents 50% of the total test score.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding choice on your answer sheet. Do not spend too much time on any one problem.

Good Luck!

NAME:

1. $\lim_{x\to\infty} \dfrac{5x^2}{3x^2+100000x} =$

A) 0

B) 0.005

C) 1

D) 1.667

E) does not exist

---

2. Which of the following functions are not differentiable at $x = \frac{2}{3}$?

I. $f(x) = \sqrt[3]{x-2}$ II. $g(x) = |3x-2|$ III. $h(x) = |9x^2-4|$

A) I only

B) II only

C) I and II only

D) II and III only

E) I and III only

---

3. If $y = (\ln x)^3$, then $dy/dx =$

A) $\frac{3}{x}(\ln x)^2$

B) $3(\ln x)^2$

C) $3x(\ln x)^2 + (\ln x)^3$

D) $3(\ln x + 1)$

E) None of these

4. If $F(x) = x \sin x$, then find $F'(3\pi/2)$.

A) 0

B) 1

C) -1

D) $3\pi/2$

E) $-3\pi/2$

5. $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

6. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at the point $(2, 1)$ is

A) $-\frac{3}{2}$

B) -1

C) $-\frac{5}{14}$

D) $-\frac{3}{14}$

E) 0

7. The average rate of change of $f(x) = x^3$ over the interval $[a, b]$ is

A) $3b + 3a$

B) $b^2 + ab + a^2$

C) $\dfrac{b^2 + a^2}{2}$

D) $\dfrac{b^3 - a^3}{2}$

E) $\dfrac{b^4 - a^4}{4(b - a)}$

---

8. The function

$$f(x) = \begin{cases} 4 - x^2 & x \leq 1 \\ mx + b & x > 1 \end{cases}$$

is continuous and differentiable for all real numbers. What must be the values of m and b ?

A) $m = 2,\ b = 1$

B) $m = 2,\ b = 5$

C) $m = -2,\ b = 1$

D) $m = -2,\ b = 5$

E) None of these

---

9. If $f(x) = -x^2 + x$, then which of the following expressions represents $f'(x)$?

A) $\lim\limits_{h \to 0} \dfrac{(-x^2 + x + h) - (-x^2 + x)}{h}$

B) $\lim\limits_{h \to x} \dfrac{(-x^2 + x + h) - (-x^2 + x)}{h}$

C) $\dfrac{\left[-(x + h)^2 + (x + h)\right] - (-x^2 + x)}{h}$

D) $\lim\limits_{h \to 0} \dfrac{\left[-(x + h)^2 + (x + h)\right] - (-x^2 + x)}{h}$

E) None of these

10. If $f(x)=\tan(2x)$, then $f'\left(\frac{\pi}{6}\right)=$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

11. If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

12. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

13. If $y=\arctan(\cos x)$, then $\frac{dy}{dx}=$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2\sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

14. $\frac{d}{dx}(\arcsin 2x)=$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

15. If $y=\cos^2 3x$, then $\frac{dy}{dx}=$

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

16. All the functions below, except one, have the property that $f(x)$ is equal to its fourth derivative, $f^{(4)}(x)$. Which one does not have this property?

A) $f(x) = \sin x$

B) $f(x) = \cos x$

C) $f(x) = -5e^x$

D) $f(x) = e^{2x}$

E) $f(x) = e^{-x}$

---

17. If $g(t) = \dfrac{\ln t}{e^t}$, then $g'(t) =$

A) $\dfrac{1 - \ln t}{e^t}$

B) $\dfrac{1 - t\ln t}{e^t}$

C) $\dfrac{t\ln t - 1}{te^t}$

D) $\dfrac{1 - t\ln t}{te^t}$

E) $\dfrac{1 - e^t\ln t}{e^{2t}}$

---

18. If $H(x) = x^3 - x^2 + \dfrac{1}{x}$, which of the following is $H''(2)$?

A) $\frac{31}{4}$

B) $\frac{39}{4}$

C) $\frac{79}{8}$

D) $\frac{81}{8}$

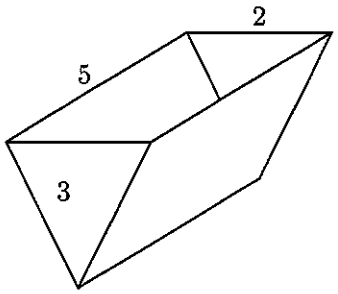
E) $\frac{41}{4}$

19. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \leq y \leq 2\pi$.

a) Find dy/dx in terms of y.

b) Write an equation for each vertical tangent to the curve.

c) Find $\dfrac{d^2y}{dx^2}$ in terms of y.



20. The trough shown in the figure above is 5 feet long and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

a) Find the volume of water when the trough is full.

b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

c) What is the rate of change in the area of the surface of the water at the instant when the trough is $\frac{1}{4}$ full by volume?

---

21. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

a) Find the domain of f.

b) Determine whether f is an odd or even function.

A. Find $f'(x)$.

B. Find the slope of the line normal to the graph of f at $x = 5$.