

AP AB Calculus

Free Response Questions

Riemann Sums

Hopkins

Years: 2003 to 2014

**2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

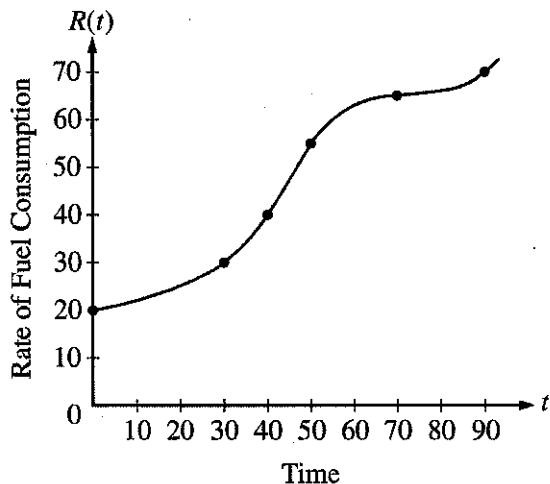
Distance $x$ (cm)	0	1	5	6	8
Temperature $T(x)$ ( $^{\circ}\text{C}$ )	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature  $T(x)$ , in degrees Celsius ( $^{\circ}\text{C}$ ), of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.
- (a) Estimate  $T'(7)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of  $T(x)$  for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find  $\int_0^8 T'(x) dx$ , and indicate units of measure. Explain the meaning of  $\int_0^8 T'(x) dx$  in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that  $T''(x) > 0$  for every  $x$  in the interval  $0 < x < 8$ ? Explain your answer.
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**WRITE ALL WORK IN THE TEST BOOKLET.**

**END OF PART A OF SECTION II**

2003 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS



$t$ (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function  $R$  of time  $t$ . The graph of  $R$  and a table of selected values of  $R(t)$ , for the time interval  $0 \leq t \leq 90$  minutes, are shown above.

- (a) Use data from the table to find an approximation for  $R'(45)$ . Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time  $t = 45$  minutes. What is the value of  $R''(45)$ ? Explain your reasoning.
- (c) Approximate the value of  $\int_0^{90} R(t) dt$  using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of  $\int_0^{90} R(t) dt$ ? Explain your reasoning.
- (d) For  $0 < b \leq 90$  minutes, explain the meaning of  $\int_0^b R(t) dt$  in terms of fuel consumption for the plane. Explain the meaning of  $\frac{1}{b} \int_0^b R(t) dt$  in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

DAY 84  
W↑

CALCULUS AB  
SECTION II, Part B  
Time—45 minutes  
Number of problems—3

No calculator is allowed for these problems.

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$t$ (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity  $v(t)$  after being launched upward from an initial height of 0 feet at time  $t = 0$  seconds. The velocity of the rocket is recorded for selected values of  $t$  over the interval  $0 \leq t \leq 80$  seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval  $0 \leq t \leq 80$  seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of  $\int_{10}^{70} v(t) dt$  in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate  $\int_{10}^{70} v(t) dt$ .
- (c) Rocket B is launched upward with an acceleration of  $a(t) = \frac{3}{\sqrt{t+1}}$  feet per second per second. At time  $t = 0$  seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time  $t = 80$  seconds? Explain your answer.
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**2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB**  
**SECTION II, Part B**  
**Time—45 minutes**  
**Number of problems—3**

**No calculator is allowed for these problems.**

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4. A particle moves along the  $x$ -axis with position at time  $t$  given by  $x(t) = e^{-t} \sin t$  for  $0 \leq t \leq 2\pi$ .
- (a) Find the time  $t$  at which the particle is farthest to the left. Justify your answer.
- (b) Find the value of the constant  $A$  for which  $x(t)$  satisfies the equation  $Ax''(t) + x'(t) + x(t) = 0$  for  $0 < t < 2\pi$ .
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$t$ (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function  $r$  of time  $t$ , where  $t$  is measured in minutes. For  $0 < t < 12$ , the graph of  $r$  is concave down. The table above gives selected values of the rate of change,  $r'(t)$ , of the radius of the balloon over the time interval  $0 \leq t \leq 12$ . The radius of the balloon is 30 feet when  $t = 5$ .
- (Note: The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .)
- (a) Estimate the radius of the balloon when  $t = 5.4$  using the tangent line approximation at  $t = 5$ . Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when  $t = 5$ . Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_0^{12} r'(t) dt$ ? Give a reason for your answer.
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**2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- ➔(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- ➔(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?
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2009 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

$x$	2	3	5	8	13
$f(x)$	1	4	-2	3	6

5. Let  $f$  be a function that is twice differentiable for all real numbers. The table above gives values of  $f$  for selected points in the closed interval  $2 \leq x \leq 13$ .

- (a) Estimate  $f'(4)$ . Show the work that leads to your answer.
- (b) Evaluate  $\int_2^{13} (3 - 5f'(x)) dx$ . Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate  $\int_2^{13} f(x) dx$ . Show the work that leads to your answer.
- (d) Suppose  $f'(5) = 3$  and  $f''(x) < 0$  for all  $x$  in the closed interval  $5 \leq x \leq 8$ . Use the line tangent to the graph of  $f$  at  $x = 5$  to show that  $f(7) \leq 4$ . Use the secant line for the graph of  $f$  on  $5 \leq x \leq 8$  to show that  $f(7) \geq \frac{4}{3}$ .

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2010 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

$t$ (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ( $t = 0$ ) and 8 P.M. ( $t = 8$ ). The number of entries in the box  $t$  hours after noon is modeled by a differentiable function  $E$  for  $0 \leq t \leq 8$ . Values of  $E(t)$ , in hundreds of entries, at various times  $t$  are shown in the table above.

→(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time  $t = 6$ . Show the computations that lead to your answer.

↘(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of  $\frac{1}{8} \int_0^8 E(t) dt$ .

Using correct units, explain the meaning of  $\frac{1}{8} \int_0^8 E(t) dt$  in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function  $P$ , where  $P(t) = t^3 - 30t^2 + 298t - 976$  hundreds of entries per hour for  $8 \leq t \leq 12$ . According to the model, how many entries had not yet been processed by midnight ( $t = 12$ )?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



2011 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

$t$ (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

2012 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB  
SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

→(a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

→(b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.

→(c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?

2014 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

$t$ (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

4. Train  $A$  runs back and forth on an east-west section of railroad track. Train  $A$ 's velocity, measured in meters per minute, is given by a differentiable function  $v_A(t)$ , where time  $t$  is measured in minutes. Selected values for  $v_A(t)$  are given in the table above.
- (a) Find the average acceleration of train  $A$  over the interval  $2 \leq t \leq 8$ .
- (b) Do the data in the table support the conclusion that train  $A$ 's velocity is  $-100$  meters per minute at some time  $t$  with  $5 < t < 8$ ? Give a reason for your answer.
- (c) At time  $t = 2$ , train  $A$ 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train  $A$ , in meters from the Origin Station, at time  $t = 12$ . Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time  $t = 12$ .
- (d) A second train, train  $B$ , travels north from the Origin Station. At time  $t$  the velocity of train  $B$  is given by  $v_B(t) = -5t^2 + 60t + 25$ , and at time  $t = 2$  the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train  $A$  and train  $B$  is changing at time  $t = 2$ .
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