

Average Value

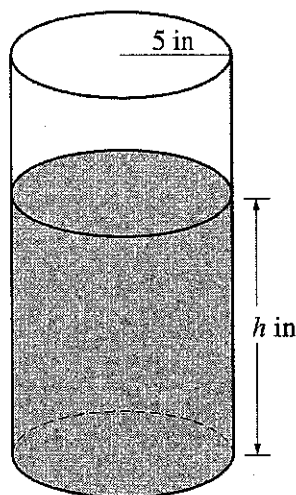
Free Response Questions

Years: 2004 – 2014

AP AB Calculus

Hopkins

2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .

- (c) At what time t is the coffeepot empty?

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is f continuous at $x = 3$? Explain why or why not.

→ (b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.

- (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx + 2 & \text{for } 3 < x \leq 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

END OF EXAMINATION

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2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4 \sin\left(\frac{t}{2}\right) \text{ for } 0 \leq t \leq 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

- (a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
- (b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
- (c) What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
- (d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
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2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.
- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

END OF PART A OF SECTION II

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

3. A particle moves along the x -axis so that its velocity v at time t , for $0 \leq t \leq 5$, is given by

$v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

- (a) Find the acceleration of the particle at time $t = 4$.
 - (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left?
 - (c) Find the position of the particle at time $t = 2$.
 - (d) Find the average speed of the particle over the interval $0 \leq t \leq 2$.
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WRITE ALL WORK IN THE TEST BOOKLET.

END OF PART A OF SECTION II

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ ($^{\circ}\text{C}$)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

→ (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

WRITE ALL WORK IN THE TEST BOOKLET.

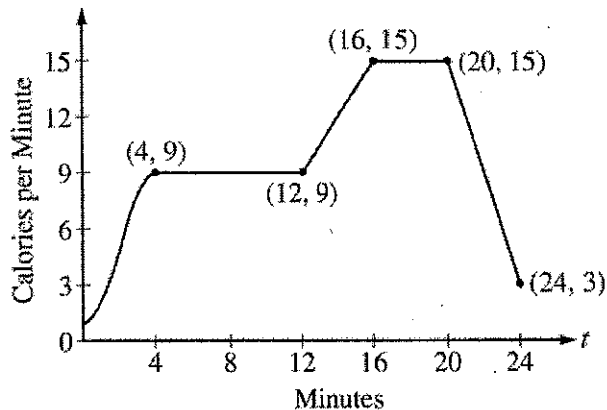
END OF PART A OF SECTION II

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part B

Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f . In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \leq t \leq 4$ and f is piecewise linear for $4 \leq t \leq 24$.
- (a) Find $f'(22)$. Indicate units of measure.
- (b) For the time interval $0 \leq t \leq 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.
- (c) Find the total number of calories burned over the time interval $6 \leq t \leq 18$ minutes.
- (d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \leq t \leq 18$.

WRITE ALL WORK IN THE EXAM BOOKLET.

2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?
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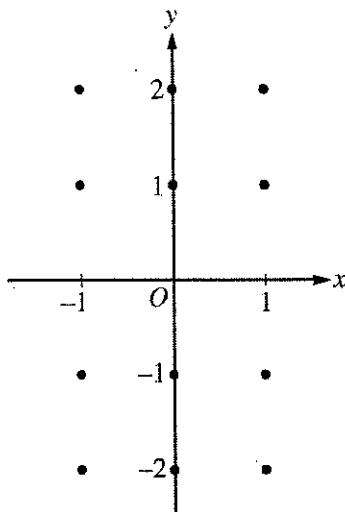
WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2010 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane for which $y \neq 0$. Describe all points in the xy -plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

6. Two particles move along the x -axis. For $0 \leq t \leq 6$, the position of particle P at time t is given by

$$p(t) = 2\cos\left(\frac{\pi}{4}t\right), \text{ while the position of particle } R \text{ at time } t \text{ is given by } r(t) = t^3 - 6t^2 + 9t + 3.$$

- (a) For $0 \leq t \leq 6$, find all times t during which particle R is moving to the right.
- (b) For $0 \leq t \leq 6$, find all times t during which the two particles travel in opposite directions.
- (c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.
- (d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \leq t \leq 3$.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

2010 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \leq t \leq 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

→ (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8} \int_0^8 E(t) dt$.

Using correct units, explain the meaning of $\frac{1}{8} \int_0^8 E(t) dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P , where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \leq t \leq 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2011 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

→ (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.

(c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

2014 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.
- * (a) Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
- (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
- (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
- (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.
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