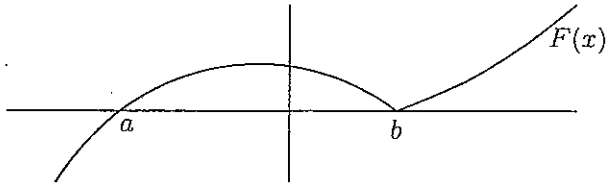
PRETEST 2014

Test

Applications of the Derivative

AP AB Calculus

Hopkins



1.

1. The function F above satisfies the conclusion of Rolle's Theorem in the interval $[a, b]$ because

I. F is continuous.
II. F is differentiable on (a, b).
III. $F(a) = F(b) = 0$.

A) I only

B) II only

C) I and III only

D) I, II, and III

E) F does not satisfy Rolle's Theorem

2.

Let f be a twice-differentiable function of x such that, when $x = c$, f is decreasing, concave up, and has an x-intercept. Which of the following is true?

A) $f(c) < f'(c) < f''(c)$

B) $f(c) < f''(c) < f'(c)$

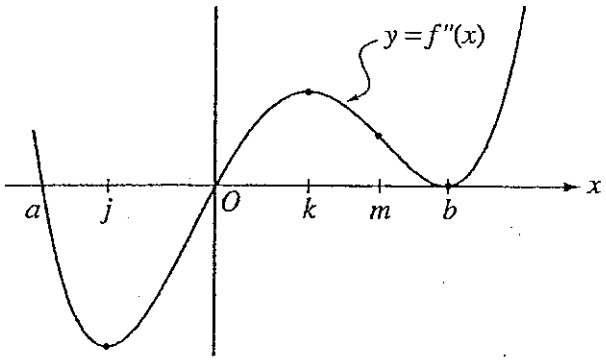
C) $f'(c) < f(c) < f''(c)$

D) $f'(c) < f''(c) < f(c)$

E) $f''(c) < f(c) < f'(c)$

3.

3.



The second derivative of the function f is given by $f''(x) = x(x - a)(x - b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k

4. The value of c guaranteed to exist by the Mean Value Theorem for $f(x) = x^2$ in the interval $[0, 3]$ is

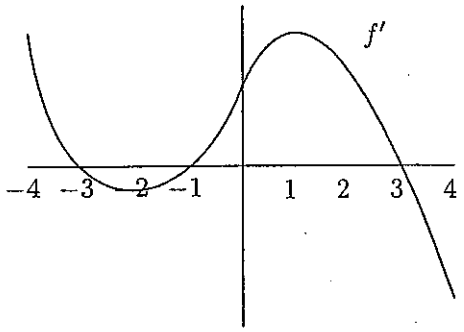
A) 1

B) 2

C) $\frac{3}{2}$

D) $\frac{1}{2}$

E) None of these



5. The graph of the derivative of a function f is shown above. Which of the following are true about the original function f ?

I. f is increasing on the interval $(-2, 1)$.
II. f is continuous at $x = 0$.
III. f has an inflection point at $x = -2$.

A) I only

B) II only

C) III only

D) II and III only

E) I, II, and III

6. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.

(B) f is decreasing.

(C) f is discontinuous.

(D) f has a relative minimum.

(E) f has a relative maximum.

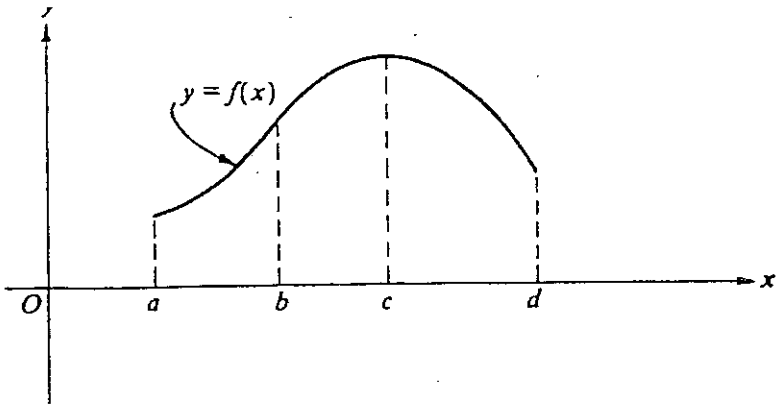
7. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) −2

8. If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

9.



The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

10. The function f given by $f(x) = x^3 + 12x - 24$ is

(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$

(B) decreasing for $x < 0$, increasing for $x > 0$

(C) increasing for all x

(D) decreasing for all x

(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

11. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

12. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\dfrac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\dfrac{1}{\sqrt{2}}\right)$

13. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

14. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

15. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

16. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

1. A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

a) Find the velocity at time t.

b) Find the acceleration at time t.

c) What are all values of t, for $0 \leq t \leq 3$, for which the particle is at rest?

d) What is the maximum velocity?

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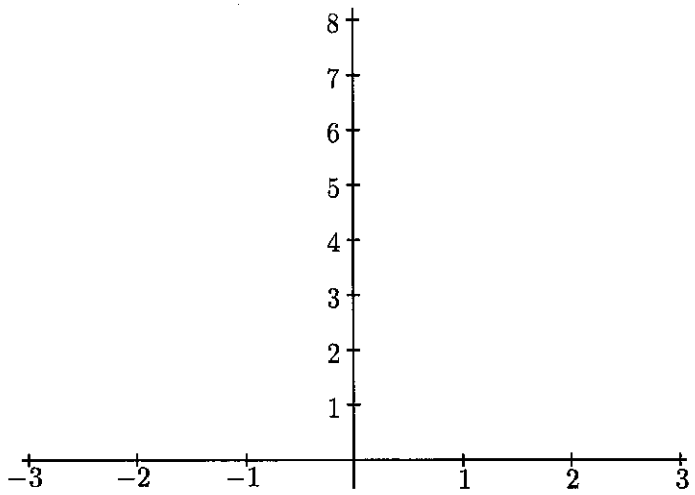
2. A function f is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The function f' and f'' have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	positive	fails to exist	negative	0	negative
$f''(x)$	positive	fails to exist	positive	0	negative

a) What are the x-coordinates of all absolute maximum and absolute minimum points of f on the interval $[-3, 3]$? Justify your answer.

b) What are the x-coordinates of all points of inflection of f on the interval $[-3, 3]$? Justify your answer.

c) On the axes provided, sketch a graph that satisfies the given properties of f.



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3. Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

a) On what intervals is f increasing?

b) On what intervals is the graph of f concave downward?

c) Find the value of k for which f has 11 as its relative minimum.