AP Calculus Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

WS 5.4 Concavity and the Second Derivative Test

Use the concavity test to determine the intervals on which the graph of the function is concave up and concave down and find all points of inflection. No Calculator allowed.

1.  2. 

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3. The graph of the derivative, , of a function *f* is shown. On what interval(s) is *f* concave up and concave down? Justify your answer.





4. The graph of the second derivative, , of a function *f* is shown. State the *x*-coordinates of the inflection points of *f*. Justify your answer.





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5. Let  be a function with a second derivative given by  . What are the x-coordinates of the points of inflection of the graph of ? No calculator allowed.

(A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, 6

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6. The derivative of the function  is given by  . How many points of inflection does the graph of  have on the open interval ? Calculator allowed.

(A) One (B) Two (C) Three (D) Four (E) Five

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7. Let  be the function given by . For what interval of x is the graph of  is concave down?

No calculator allowed.

(A)  (B)  (C)  (D)  (E) 

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8. The graph of a differentiable function  is shown. Which of the following is true?

(A) 

(B) 

(C) 

(D) 

(E) 

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9. For  , let  be a function with first derivative given by  . Which of the following are all intervals in which the graph of f is concave down? Calculator allowed.

(A)  only

(B) 

(C) and 

(D)  and 

(E)  ,  , and 

Find the critical points of each function, and determine whether they are relative maximums or relative minimums by using the Second Derivative Test. No calculator allowed.

10. 

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11. Suppose that the function *f* has a continuous second derivative for all *x* and that . Let *g* be a function whose derivative is given by  for all *x*. Does *g* have a local maximum or a local minimum at ? Justify your answer.

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12. Consider the curve given by  and  Find the value of  at the point *P* (3, 2). Does the curve have a local maximum, a local minimum, or neither at point *P*? Justify your answer.

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13. Let f(x) be a twice differentiable function defined for all real numbers. f(x) is not constant. The table gives values of the first and second derivative at several points. The function has no critical points other than those mentioned in the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***x*** | **-3** | **-2** | **2** | **4** |
| ***f'(x)*** | 1 | 0 | 1 | 0 |
| ***f"(x)*** | 2 | -3 | 0 | 1 |

a) Does the function have a maximum or a minimum when x = -2? Justify your answer.

b) Does the function have a maximum or a minimum when x = 4? Justify your answer.

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14. Let  be a polynomial function with values of  at selected values of x in the given table. Which of the following must be true for ?

(A) The graph of  is concave up.

(B) The graph of  has at least two points of inflection.

(C)  is increasing.

(D)  has no critical points.

(E)  has at least two relative extrema.

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15. The graph of the function  is shown in the figure.

For which of the following values of x is  positive and increasing?

(A) a (B) b

(C) c (D) d (E) e