

AP Calculus

Day 61 Points of Inflection and Concavity

Name \_\_\_\_\_

Date \_\_\_\_\_

1. Find all points of inflection:  $f(x) = x^3 - 12x$

- a)  $(0, 0)$ ,  $(\pm\sqrt{12}, 0)$       b)  $(0, 0)$   
 c)  $(2, 0)$ ,  $(-2, 0)$       d)  $(2, -16)$ ,  $(-2, 16)$   
 e)  $(0, 0)$ ,  $(2, -16)$

2. If  $f(x) = x^3 + 6x^2 - 5x + 10$ , determine its point of inflection.

- a)  $(-2, 36)$       b)  $(2, 32)$       c)  $(0, 10)$   
 d)  $(2, 31)$       e)  $(-2, -17)$

3. For the function  $f(x) = x^4 - 4x^3 + 2$ , determine all points of inflection.

- a)  $(0, 2)$  and  $(2, -14)$   
 b)  $(0, -14)$  and  $(12, 13,826)$   
 c)  $(0, 2)$  and  $(4, 2)$   
 d)  $(-12, 27,650)$  and  $(4, 2)$   
 e)  $(12, 13,826)$  and  $(-12, 27,650)$

4. Let  $f$  be defined by  $f(x) = x + 1 - (\cos x)^2$ . Find the  $x$ -coordinates of all inflection points of  $f$ ,  $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$ .

- a)  $\frac{\pi}{4}$       b)  $\frac{\pi}{2}$       c)  $\frac{\pi}{4}$ ,  $\frac{3\pi}{4}$   
 d)  $\frac{\pi}{2}$ ,  $\frac{3\pi}{4}$       e)  $\frac{\pi}{2}$ ,  $\frac{5\pi}{6}$

5. Given  $f(x) = (1 + \ln x)^3$  where  $x \geq 0$ , find the points of inflection on the graph of  $f(x)$ .

- a)  $(e, 8)$  only      b)  $(\frac{1}{e}, 8)$  only  
 c)  $(e, 8)$ ,  $(\frac{1}{e}, 8)$       d)  $(e, 8)$ ,  $(\frac{1}{e}, 0)$   
 e)  $(0, 8)$ ,  $(\frac{1}{e}, e)$

6. Given a curve is defined by the equation  $f(x) = (1 - \ln x)^2$ . Find a point of inflection.

- a)  $(2e, e)$       b)  $(1, 0)$       c)  $(2e^2, 1)$   
 d)  $(e^{-2}, e)$       e)  $(e^2, 1)$

7. The graph  $y = \frac{2}{3}ax^3 + 3ax^2 + 4ax + 2$ ,  $a \in I$ , has a point of inflection at  $x =$

- a) 3      b)  $-\frac{3}{2}$       c)  $a$       d)  $a^2$       e)  $\frac{3}{a}$

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8. Given  $f(x) = \frac{3x^4 - 256}{3x^2}$ . What are the  $x$ -coordinates of the points of inflection for the graph of  $f$ ?
- a)  $\emptyset$                       b)  $-4$  only                      c)  $4$  only  
d)  $\pm 4$                       e)  $\pm 4\sqrt{2}$
9. Find all intervals on which the function  $y = 8x^3 - 2x^4$  is concave downward.
- a)  $(-\infty, 0)$  and  $(2, \infty)$   
b)  $(-\infty, 24)$  and  $(48, \infty)$   
c)  $(-\infty, 2)$  and  $(8, \infty)$   
d)  $(0, 2)$   
e)  $(24, 48)$
10. Find all intervals on which the function  $y = 8x^3 - 2x^4$  is concave upward.
- a)  $(-\infty, 0)$  and  $(2, \infty)$   
b)  $(-\infty, 24)$  and  $(48, \infty)$   
c)  $(-\infty, 2)$  and  $(8, \infty)$   
d)  $(0, 2)$   
e)  $(24, 48)$
11. Find all intervals on which the function  $y = 6x^3 - x^4$  is concave upward.
- a)  $(-\infty, 0)$  and  $(3, \infty)$     b)  $(-\infty, 0)$  and  $(6, \infty)$   
c)  $(-\infty, 3)$  and  $(6, \infty)$     d)  $(0, 3)$   
e)  $(3, 6)$
12. Let  $f$  be defined by  $f(x) = (x^2 - 1)^3$  for all real numbers  $x$ . Find a region where the graph of  $f$  is concave downward.
- a)  $(-\infty, 0)$                       b)  $(-1, 1)$                       c)  $(0, 1)$   
d)  $(-\infty, -1)$                       e)  $(\sqrt{\frac{1}{5}}, 1)$
13. Find a region where the graph of  $f$  is concave downward, if  $f$  be defined by  $f(x) = (x^2 - 4)^3$  for all real numbers  $x$ .
- a)  $(-\infty, 2)$                       b)  $(-2, 2)$                       c)  $(\frac{4}{5}, 2)$   
d)  $(2, \infty)$                       e)  $(\frac{2}{\sqrt{5}}, 2)$

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14. Given  $f(x) = 2x^4(x^2 - 3)$ . For what value(s) is the graph of  $f$  concave upwards?

- a)  $0 < x < \sqrt{\frac{6}{5}}$
- b)  $-\sqrt{\frac{6}{5}} < x < 0$
- c)  $-\infty < x < -\sqrt{\frac{6}{5}}$  and  $\sqrt{\frac{6}{5}} < x < \infty$
- d)  $-\sqrt{\frac{6}{5}} < x < \sqrt{\frac{6}{5}}$
- e)  $\emptyset$

15. Find the interval(s) on which the curve  $y = 27x^3 + 27x^2 + 9x + 1$  is concave upward or concave downward.

- a) CD  $(-\frac{1}{3}, \infty)$
- b) CU  $(\frac{1}{3}, \infty)$
- c) CD  $(-\infty, \frac{1}{3})$ ; CU  $(\frac{1}{3}, \infty)$
- d) CD  $(-\frac{1}{3}, \infty)$ ; CU  $(-\infty, -\frac{1}{3})$
- e) CD  $(-\infty, -\frac{1}{3})$ ; CU  $(-\frac{1}{3}, \infty)$

16. If  $y = 3 + \sqrt[5]{x}$ , find the interval(s) on which the curve is concave upward or concave downward.

- a) CU  $(-\infty, 0)$
- b) CD  $(0, \infty)$
- c) CD  $(0, \infty)$ ; CU  $(-\infty, 0)$
- d) CD  $(-\infty, 0)$ ; CU  $(0, \infty)$
- e) CD  $(-\infty, 1)$ ; CU  $(1, \infty)$

17. Given that  $f(x) = \frac{-8}{x}$ , choose the correct statement.

- a)  $f$  is concave up on the interval  $(0, \infty)$
- b)  $f$  is concave up on the interval  $(-\infty, 0)$
- c)  $f$  is concave down on the interval  $(-\infty, 0)$
- d)  $f$  is concave up on the interval  $(-\infty, \infty)$
- e)  $f$  is concave down for all real values

18. The graph of  $f(x) = \frac{-7}{x-4}$  is concave downward for all values of  $x$  such that

- a)  $x < 4$
- b)  $x > 4$
- c)  $x < 0$
- d)  $x > 0$
- e)  $x > -7$

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19. Determine the intervals on which the following is concave downward:  $f(x) = \frac{x-1}{x+3}$ .

- a)  $(-\infty, \infty)$     b)  $(-\infty, -3)$     c)  $(1, \infty)$   
d)  $(-3, \infty)$     e)  $(-3, 1)$

20. Given that  $f$  is continuous and the following information:

Intervals	$x < 0$	$0 < x < 3$	$3 < x$
Sign of $f'$	-	+	-

- a) For what  $x$ -value(s) is there a local minimum?  
b) For what  $x$ -value(s) is there a local maximum?

1.  
Answer:        b  
CodePath:    EAS.APC.D.J.1
2.  
Answer:        a  
CodePath:    EAS.APC.D.J.4
3.  
Answer:        a  
CodePath:    EAS.APC.D.J.7
4.  
Answer:        c  
CodePath:    EAS.APC.D.J.11
5.  
Answer:        d  
CodePath:    EAS.APC.D.J.13
6.  
Answer:        e  
CodePath:    EAS.APC.D.J.14
7.  
Answer:        b  
CodePath:    EAS.APC.D.J.18
8.  
Answer:        d  
CodePath:    EAS.APC.D.J.20
9.  
Answer:        a  
CodePath:    EAS.APC.D.J.23
10.  
Answer:        d  
CodePath:    EAS.APC.D.J.24
11.  
Answer:        d  
CodePath:    EAS.APC.D.J.26
12.  
Answer:        e  
CodePath:    EAS.APC.D.J.28
13.  
Answer:        e  
CodePath:    EAS.APC.D.J.30
14.  
Answer:        c  
CodePath:    EAS.APC.D.J.31

15.  
Answer:        e  
CodePath:    EAS.APC.D.J.34
16.  
Answer:        c  
CodePath:    EAS.APC.D.J.36
17.  
Answer:        b  
CodePath:    EAS.APC.D.J.40
18.  
Answer:        b  
CodePath:    EAS.APC.D.J.43
19.  
Answer:        d  
CodePath:    EAS.APC.D.J.47
20.  
Answer:        0, 3  
CodePath:    EAS.APC.D.E.50