

Solutions

Day 47

Warm Up

Free Response #6 2011

{ Almost, "e"
letter is different }

$$\text{Let } f(x) = \begin{cases} 1 - \sin x, & x \leq 0 \\ e^{-4x}, & x > 0 \end{cases}$$

A. Show that f is continuous at $x = 0$.

$$1 - \sin 0 \stackrel{?}{=} e^{-4(0)}$$
$$1 = 1$$

yes the continuous b/c

$$f(0) \text{ exists } \checkmark$$
$$\lim_{x \rightarrow 0} \text{ exists } \checkmark$$
$$f(0) = \lim_{x \rightarrow 0} \checkmark$$

B. Express $f'(x)$ as a piecewise function of x . Find the value of x for which $f'(x) = -3$.

$$f'(x) = \begin{cases} -\cos x, & x \leq 0 \\ -4e^{-4x}, & x > 0 \end{cases}$$
$$x = \frac{\ln \frac{3}{4}}{-4}$$

$$-\cos x = -3$$
$$\cos x = 3$$
$$\emptyset$$
$$-4e^{-4x} = -3$$
$$\ln e^{-4x} = \ln \frac{3}{4}$$
$$-4x = \ln \frac{3}{4}$$

C. Find the instantaneous rate of change of the function at $x = 5$. Find the average rate of change of the function over the interval $[1, 5]$

A. $-4e^{-4(5)} = \frac{-4}{e^{20}}$

B. $\frac{f(5) - f(1)}{5 - 1} = \frac{e^{-20} - e^{-4}}{4}$