CLASSWORK

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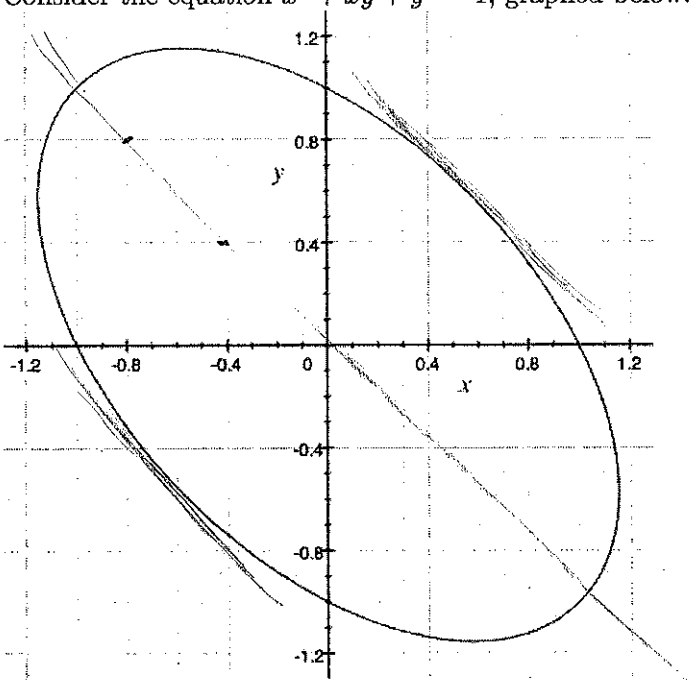
Math 1110: In-Class Problems for 3.7

Problem 1

Consider the equation $x^2 + xy + y^2 = 1$, graphed below.



(a) Find the equation for $\frac{dy}{dx}$ in terms of x and y.

$$2x + xy' + y + 2yy' = 0$$

$$y'(x + 2y) = -2x - y$$

$$\boxed{y' = \frac{-2x - y}{x + 2y}}$$

(b) Find all points (a, b) on the curve where the tangent line is parallel to the line $y = -x$.

$$\frac{-2x - y}{x + 2y} = -1$$

$$-2x - y = -x - 2y$$

$$-x = -y$$

$$x = y$$

$$x^2 + x^2 + x^2 = 1$$

$$3x^2 = 1$$

$$\boxed{x = \pm\sqrt{\frac{1}{3}}}$$

$\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(c) Find all points (c, d) on the curve where the normal line (**NOT the tangent!**) is horizontal.

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Problem 2 Consider the curve defined by $xy^2 + 4y - 10 = 2x$

(a) Find the slope of this curve at the point $(1, 2)$.

$x2yy' + y^2 + 4y' = 2$

$4y' + 4 + 4y' = 2$

$8y' = -2$

$y' = -\frac{1}{4}$

(b) Find the equation of the normal to the curve at the point $(1, 2)$

"normal" means $\perp$

So if $m = -\frac{1}{4}$

$\perp m = 4$

$y - 2 = m(x - 1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y = 4x - 2$

Problem 3 Calculate dy/dx if

(a) $x\cos(y) = y\cos(x)$

$x(-\sin y)y' + \cos(y)(1) = y(-\sin x) + \cos(x)\,y'$

$\cos y + y\sin x = y'\cos x + y'x\sin y \Rightarrow$

$y' = \frac{\cos y + y\sin x}{\cos x + x\sin y}$

(b) $e^x = \cos(x - y)$

$e^x = -\sin(x-y)(1 - y')$

$e^x = -\sin(x-y) + y'\sin(x-y)$

$e^x + \sin(x-y) = y'(\sin(x-y))$

$\frac{e^x + \sin(x-y)}{\sin(x-y)} = y'$

(c) $y = \sin(xy)$

$y' = \cos(xy)(xy' + y)$

$y' = xy'\cos(xy) + y\cos(xy)$

$y' - xy'\cos(xy) = y\cos(xy)$

$y'(1 - x\cos(xy)) = y\cos(xy)$

$y' = \frac{y\cos(xy)}{1 - x\cos(xy)}$