Exponential Derivatives Wksht.

3.7 EXERCISES

Preliminary Questions

1. Identify the outside and inside functions for each of these composite functions.

(a) $y = \sqrt{4x + 9x^2}$ (b) $y = \tan(x^2 + 1)$

(c) $y = \sec^5 x$ (d) $y = (1 + e^x)^4$

2. Which of the following can be differentiated easily *without* using the Chain Rule?

(a) $y = \tan(7x^2 + 2)$ (b) $y = \dfrac{x}{x+1}$

(c) $y = \sqrt{x} \cdot \sec x$ (d) $y = \sqrt{x} \cos x$

(e) $y = xe^x$ (f) $y = e^{\sin x}$

3. Which is the derivative of $f(5x)$?

(a) $5f'(x)$ (b) $5f'(5x)$ (c) $f'(5x)$

4. Suppose that $f'(4) = g(4) = g'(4) = 1$. Do we have enough information to compute $F'(4)$, where $F(x) = f(g(x))$? If not, what is missing?

Exercises

In Exercises 1–4, fill in a table of the following type:

$f(g(x))$	$f'(u)$	$f'(g(x))$	$g'(x)$	$(f \circ g)'$

1. $f(u) = u^{3/2}$, $g(x) = x^4 + 1$

2. $f(u) = u^3$, $g(x) = 3x + 5$

3. $f(u) = \tan u$, $g(x) = x^4$

4. $f(u) = u^4 + u$, $g(x) = \cos x$

In Exercises 5 and 6, write the function as a composite $f(g(x))$ and compute the derivative using the Chain Rule.

5. $y = (x + \sin x)^4$ 6. $y = \cos(x^3)$

7. Calculate $\dfrac{d}{dx} \cos u$ for the following choices of $u(x)$:

(a) $u = 9 - x^2$ (b) $u = x^{-1}$ (c) $u = \tan x$

8. Calculate $\dfrac{d}{dx} f(x^2 + 1)$ for the following choices of $f(u)$:

(a) $f(u) = \sin u$ (b) $f(u) = 3u^{3/2}$ (c) $f(u) = u^2 - u$

9. Compute $\dfrac{df}{dx}$ if $\dfrac{df}{du} = 2$ and $\dfrac{du}{dx} = 6$.

10. Compute $\left.\dfrac{df}{dx}\right|_{x=2}$ if $f(u) = u^2$, $u(2) = -5$, and $u'(2) = -5$.

In Exercises 11–22, use the General Power Rule or the Shifting and Scaling Rule to compute the derivative.

11. $y = (x^4 + 5)^3$ 12. $y = (8x^4 + 5)^3$

13. $y = \sqrt{7x - 3}$ 14. $y = (4 - 2x - 3x^2)^5$

15. $y = (x^2 + 9x)^{-2}$ 16. $y = (x^3 + 3x + 9)^{-4/3}$

17. $y = \cos^4 \theta$ 18. $y = \cos(9\theta + 41)$

19. $y = (2\cos\theta + 5\sin\theta)^9$ 20. $y = \sqrt{9 + x + \sin x}$

21. $y = e^{x-12}$ 22. $y = e^{8x+9}$

In Exercises 23–26, compute the derivative of $f \circ g$.

23. $f(u) = \sin u$, $g(x) = 2x + 1$

24. $f(u) = 2u + 1$, $g(x) = \sin x$

25. $f(u) = e^u$, $g(x) = x + x^{-1}$

26. $f(u) = \dfrac{u}{u-1}$, $g(x) = \csc x$

In Exercises 27 and 28, find the derivatives of $f(g(x))$ and $g(f(x))$.

27. $f(u) = \cos u$, $u = g(x) = x^2 + 1$

28. $f(u) = u^3$, $u = g(x) = \dfrac{1}{x+1}$

In Exercises 29–42, use the Chain Rule to find the derivative.

29. $y = \sin(x^2)$ 30. $y = \sin^2 x$

31. $y = \sqrt{t^2 + 9}$ 32. $y = (t^2 + 3t + 1)^{-5/2}$

33. $y = (x^4 - x^3 - 1)^{2/3}$ 34. $y = (\sqrt{x+1} - 1)^{3/2}$

35. $y = \left(\dfrac{x+1}{x-1}\right)^4$ 36. $y = \cos^3(12\theta)$

37. $y = \sec \dfrac{1}{x}$ 38. $y = \tan(\theta^2 - 4\theta)$

39. $y = \tan(\theta + \cos\theta)$ 40. $y = e^{2x^2}$

41. $y = e^{2-9t^2}$ 42. $y = \cos^3(e^{4\theta})$

In Exercises 43–72, find the derivative using the appropriate rule or combination of rules.

43. $y = \tan(x^2 + 4x)$ 44. $y = \sin(x^2 + 4x)$

45. $y = x\cos(1 - 3x)$ 46. $y = \sin(x^2)\cos(x^2)$

47. $y = (4t + 9)^{1/2}$ 48. $y = (z + 1)^4(2z - 1)^3$

49. $y = (x^3 + \cos x)^{-4}$ 50. $y = \sin(\cos(\sin x))$

51. $y = \sqrt{\sin x \cos x}$ 52. $y = (9 - (5 - 2x^4)^7)^3$

53. $y = (\cos 6x + \sin x^2)^{1/2}$

54. $y = \dfrac{(x+1)^{1/2}}{x+2}$

55. $y = \tan^3 x + \tan(x^3)$

56. $y = \sqrt{4 - 3\cos x}$

57. $y = \sqrt{\dfrac{z+1}{z-1}}$

58. $y = (\cos^3 x + 3\cos x + 7)^9$

59. $y = \dfrac{\cos(1+x)}{1+\cos x}$

60. $y = \sec(\sqrt{t^2 - 9})$

61. $y = \cot^7(x^5)$

62. $y = \dfrac{\cos(1/x)}{1+x^2}$

63. $y = \left(1 + \cot^5(x^4+1)\right)^9$

64. $y = 4e^{-x} + 7e^{-2x}$

65. $y = (2e^{3x} + 3e^{-2x})^4$

66. $y = \cos(te^{-2t})$

67. $y = e^{(x^2+2x+3)^2}$

68. $y = e^{e^x}$

69. $y = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$

70. $y = \sqrt{\sqrt{x+1} + 1}$

71. $y = (kx+b)^{-1/3}$; k and b any constants

72. $y = \dfrac{1}{\sqrt{kt^4 + b}}$; k, b constants, not both zero

In Exercises 73–76, compute the higher derivative.

73. $\dfrac{d^2}{dx^2} \sin(x^2)$

74. $\dfrac{d^2}{dx^2} (x^2+9)^5$

75. $\dfrac{d^3}{dx^3} (9-x)^8$

76. $\dfrac{d^3}{dx^3} \sin(2x)$

77. The average molecular velocity v of a gas in a certain container is given by $v = 29\sqrt{T}$ m/s, where T is the temperature in kelvins. The temperature is related to the pressure (in atmospheres) by $T = 200P$. Find $\left.\dfrac{dv}{dP}\right|_{P=1.5}$.

78. The power P in a circuit is $P = Ri^2$, where R is the resistance and i is the current. Find dP/dt at $t = \frac{1}{3}$ if $R = 1000\ \Omega$ and i varies according to $i = \sin(4\pi t)$ (time in seconds).

79. An expanding sphere has radius $r = 0.4t$ cm at time t (in seconds). Let V be the sphere's volume. Find dV/dt when (a) $r = 3$ and (b) $t = 3$.

80. A 2005 study by the Fisheries Research Services in Aberdeen, Scotland, suggests that the average length of the species *Clupea harengus* (Atlantic herring) as a function of age t (in years) can be modeled by $L(t) = 32(1 - e^{-0.37t})$ cm for $0 \le t \le 13$. See Figure 2.

(a) How fast is the average length changing at age $t = 6$ years?

(b) At what age is the average length changing at a rate of 5 cm/yr?

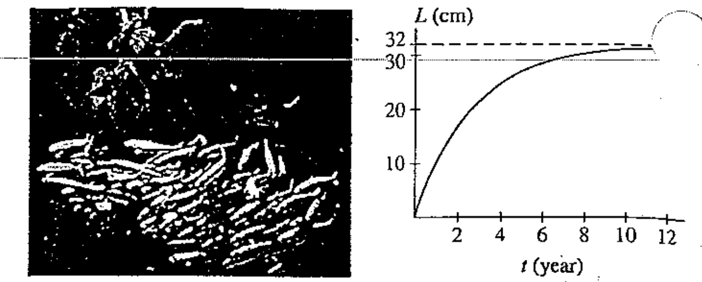

FIGURE 2 Average length of the species *Clupea harengus*

81. A 1999 study by Starkey and Scarnecchia developed the following model for the average weight (in kilograms) at age t (in years) of channel catfish in the Lower Yellowstone River (Figure 3):

$$W(t) = (3.46293 - 3.32173e^{-0.03456t})^{3.4026}$$

Find the rate at which average weight is changing at age $t = 10$.

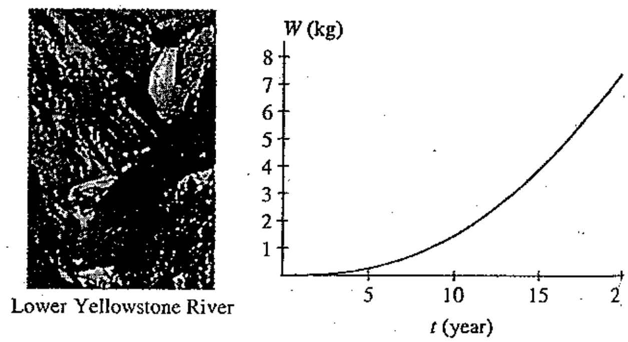

FIGURE 3 Average weight of channel catfish at age t

82. The functions in Exercises 80 and 81 are examples of the von **Bertalanffy growth function**

$$M(t) = \left(a + (b-a)e^{kmt}\right)^{1/m} \qquad (m \ne 0)$$

introduced in the 1930s by Austrian-born biologist Karl Ludwig von Bertalanffy. Calculate $M'(0)$ in terms of the constants a, b, k and m.

83. With notation as in Example 7, calculate

(a) $\left.\dfrac{d}{d\theta} \sin\theta\right|_{\theta=60°}$

(b) $\left.\dfrac{d}{d\theta} (\theta + \tan\theta)\right|_{\theta=45°}$

84. Assume that

$$f(0) = 2, \quad f'(0) = 3, \quad h(0) = -1, \quad h'(0) = 7$$

Calculate the derivatives of the following functions at $x = 0$:

(a) $(f(x))^3$ (b) $f(7x)$ (c) $f(4x)h(5x)$

85. Compute the derivative of $h(\sin x)$ at $x = \frac{\pi}{6}$, assuming that $h'(0.5) = 10$.

86. Let $F(x) = f(g(x))$, where the graphs of f and g are shown in Figure 4. Estimate $g'(2)$ and $f'(g(2))$ and compute $F'(2)$.