Math 180 Chapter 3 Practice Test

Find the derivative.

1. $f(x) = e^{(3x^2+2x+9)}$

2. $f(x) = \ln(x^3 + 2x + 18)$.

3. $f(x) = \sin^4(x^2 + 2)$.

4. $f(x) = \dfrac{\cos x}{x^2 + 1}$

5. $f(x) = (4x^3 - 7x^2)\cos x$

6. Use implicit differentiation to find dy/dx.

(a) $xy^4 + x^2y = x + 3y$

(b) $\sin(xy) = x^2 - y^2$

7. Find the equation of the tangent line at $(-1, 2)$ to the curve given by

$$x^2 + xy^2 + y^3 = 5$$

8. Suppose that $F(x) = f(x)g(x)$ and $h(x) = f(g(x))$, where

- $f(2) = 4$
- $g(2) = 3$
- $g'(2) = 3$
- $f'(2) = 1$
- $f'(3) = 5$

Find

(a) $F'(2)$

(b) $h'(2)$

9. Inverse Trig Functions:
Formulas:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{x^2+1}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

Find the derivative of each of the following functions.

(a) $y = \arctan \sqrt{x}$

(b) $y = x \arcsin(x) + \sqrt{1 - x^2}$

10. Find the derivative of each of the following functions.

(a) $y = \ln(x^2 + 3)$

(b) $y = x \ln x$

(c) $y = \ln \dfrac{\sqrt{x^2 + 5}}{x + 1}$

11. Find the derivative using logarithmic differentiation.

$$f(x) = \frac{(1-x)^2\sqrt{3x+1}}{(x+3)^2}$$

Use logarithmic differentiation to find the derivative.

12. $y = x^{\tan x}$

13. A ladder 13 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 ft from the wall?

Formulas

- Linearization of f at a: $\quad L(x) = f(a) + f'(a)(x - a)$
- Differential: $dy = \left(\dfrac{dy}{dx}\right)\, dx$

14. Find the linearization $L(x)$ of the function at a.

$$f(x) = \ln x, \quad a = 1$$

15. (a) Find the differential dy and (b) evaluate dy for the given values of x and dx.

$$y = 1/(x+1), x = 1,\ dx = -0.01$$

16. *Hyperbolic Functions*:

Formulas

$\frac{d}{dx}(\sinh x) = \cosh x$ $\qquad$ $\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\text{sech } x) = -\text{msech } x \tanh x$ $\qquad$ $\frac{d}{dx}(\text{csch } x) = -\text{csch } x \coth x$

$\frac{d}{dx}(\tanh x) = \text{sech}^2 x$ $\qquad$ $\frac{d}{dx}(\coth x) = -\text{csch}^2 x$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}(\coth^{-1} x) = -\frac{1}{1-x^2}$$

$$\frac{d}{dx}(\text{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}} \qquad \frac{d}{dx}(\text{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

Calculate y'.

(a) $y = \ln(\cosh x)$

(b) $y = \tanh^{-1}(\sqrt{x})$

Math 180 Chapter 3 Practice Test Solutions

Find the derivative.

1. $f(x) = e^{(3x^2+2x+9)}$

SOLUTION

$$f'(x) = (6x+2)e^{(3x^2+2x+9)}$$

2. $f(x) = \ln(x^3 + 2x + 18)$.

SOLUTION

$$f'(x) = \frac{3x^2+2}{x^3+2x+18}$$

3. $f(x) = \sin^4(x^2+2)$.

SOLUTION

$$f(x) = \Big(\sin(x^2+2)\Big)^4$$

$$\begin{aligned} f'(x) &= 4\Big(\sin(x^2+2)\Big)^3 \cos(x^2+2)\cdot 2x \\ &= 8x\sin^3(x^2+2)\cos(x^2+2) \end{aligned}$$

4. $f(x) = \dfrac{\cos x}{x^2+1}$

SOLUTION

$$\begin{aligned} f'(x) &= \frac{(\cos x)'(x^2+1) - (\cos x)(x^2+1)'}{(x^2+1)^2} \\ &= \frac{(-\sin x)(x^2+1) - (\cos x)(2x)}{(x^2+1)^2} \\ &= \frac{-(x^2+1)\sin x - 2x\cos x}{(x^2+1)^2} \end{aligned}$$

5. $f(x) = (4x^3 - 7x^2)\cos x$

SOLUTION

$$\begin{aligned} f'(x) &= (12x^2 - 14x)\cos x + (14x^3 - 7x^2)(-\sin x) \\ &= (12x^2 - 14x)\cos x - (14x^3 - 7x^2)\sin x \end{aligned}$$

6. Use implicit differentiation to find dy/dx.

   (a) $xy^4 + x^2y = x + 3y$

   SOLUTION

$$y' = \frac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}$$

   (b) $\sin(xy) = x^2 - y^2$

   SOLUTION

$$y' = \frac{2x - y\cos(xy)}{x\cos(xy) + \not{1} + 2y}$$

7. Find the equation of the tangent line at $(-1, 2)$ to the curve given by

$$x^2 + xy^2 + y^3 = 5$$

SOLUTION

$$(x^2)' + [(x)'y^2 + x(y^2)'] + (y^3)' = (5)'$$

$$2x + [1 \cdot y^2 + x \cdot 2yy'] + 3y^2y' = 0$$

Substitute $x = -1$, $y = 2$.

$$2(-1) + (2)^2 + (-1)(2)(2)y' + 3(2)^2y' = 0$$

$$-2+4-4y'+12y'=0$$
$$8y'=-2$$
$$y'=-\frac{1}{4}$$

We can now substitute into the equation of a line $x_1=-1$, $y_1=2$, and $m=-\frac{1}{4}$.

$$y-y_1=m(x-x_1)$$
$$y-2=-\frac{1}{4}\left(x-(-1)\right)$$
$$y=-\frac{1}{4}x-\frac{1}{4}+2$$
$$y=-\frac{x}{4}+\frac{7}{4}$$

8. Suppose that $F(x)=f(x)g(x)$ and $h(x)=f\left(g(x)\right)$, where

- $f(2)=4$
- $g(2)=3$
- $g'(2)=3$
- $f'(2)=1$
- $f'(3)=5$

Find

(a) $F'(2)$

SOLUTION

$$F'(x)=\left(f(x)g(x)\right)'=f'(x)g(x)+f(x)g'(x)$$

$$F'(2)=f'(2)g(2)+f(2)g'(2=(1)(3)+(4)(3)=15$$

(b) $h'(2)$

Solution

$$h'(x) = f'(g(x))g'(x)$$

$$\begin{aligned} h'(2) &= f'(g(2))g'(2) \\ &= f'(3)g'(2) \\ &= 5 \cdot 3 \\ &= 15 \end{aligned}$$

9. Find the derivative of each of the following functions.

(a) $y = \arctan \sqrt{x}$

Solution

$$\begin{aligned} y &= \arctan x^{1/2} \\ y' &= \left(\frac{1}{1+(x^{1/2})^2}\right) \cdot \left(\frac{1}{2}x^{-1/2}\right) \\ y' &= \frac{1}{2\sqrt{x}(1+x)} \end{aligned}$$

(b) $y = x \arcsin(x) + \sqrt{1-x^2}$

Solution

$$\begin{aligned} y &= x \arcsin(x) + (1-x^2)^{1/2} \\ y' &= ((x)'(\arcsin x) + (x)(\arcsin x)') + \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) \\ y' &= \Big((1)(\arcsin(x)) + (x)(1/\sqrt{1-x^2})\Big) - x/\sqrt{1-x^2} \\ y' &= \arcsin(x) \end{aligned}$$

10. Find the derivative of each of the following functions.

(a) $y = \ln(x^2 + 3)$
SOLUTION

$$\begin{aligned} y &= \ln(x^2+1) \\ y' &= \left(\frac{1}{x^2+1}\right) \cdot (2x) \\ y' &= \frac{2x}{x^2+1} \end{aligned}$$

(b) $y = x \ln x$
SOLUTION

$$\begin{aligned} y &= y = x \ln x \\ y' &= (x)'(\ln x) + (x)(\ln x)' \\ y' &= (1)(\ln x) + (x)(1/x) \\ y' &= \ln(x) + 1 \end{aligned}$$

(c) $y = \ln \dfrac{\sqrt{x^2+5}}{x+1}$
SOLUTION

$$\begin{aligned} y &= \ln \frac{\sqrt{x^2+5}}{x+1} \\ y &= \frac{1}{2}\ln(x^2+5) - \ln(x+1) \\ y' &= \frac{1}{2}\left(\frac{1}{x^2+5}\right) \cdot (2x) - \frac{1}{x+1} \\ y' &= \frac{x}{x^2+5} - \frac{1}{x+1} \end{aligned}$$

11. Find the derivative using logarithmic differentiation.

$$f(x) = \frac{(1-x)^2\sqrt{3x+1}}{(x+3)^2}$$

SOLUTION

$$\ln y = \ln \frac{(1-x)^2\sqrt{3x+1}}{(x+3)^2}$$

$$\ln y = 2\ln(1-x) + \frac{1}{2}\ln(3x+1) - 2\ln(x+3)$$

$$\begin{aligned}
\frac{1}{y}y' &= 2\frac{1}{1-x}\cdot(-1) + \frac{1}{2}\cdot\frac{1}{3x+1}\cdot(3) - 2\frac{1}{x+3}\cdot(1) \\
y' &= y\left[2\frac{1}{1-x})\cdot(-1) + \frac{1}{2}\frac{1}{3x+1}\cdot(3) - 2\cdot\frac{1}{x+3}\cdot(1)\right] \\
y' &= \frac{(1-x)^2\sqrt{3x+1}}{(x+3)^2}\left[-\frac{2}{1-x} + \frac{3}{2}\cdot\frac{1}{3x+1} - \frac{2}{x+3}\right]
\end{aligned}$$

Use logarithmic differentiation to find the derivative.

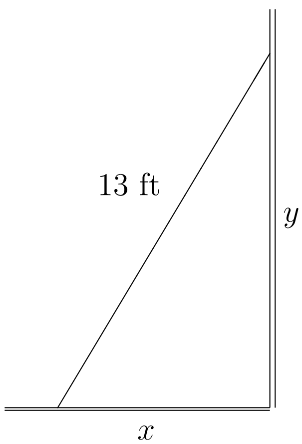
12. $y = x^{\tan x}$

SOLUTION

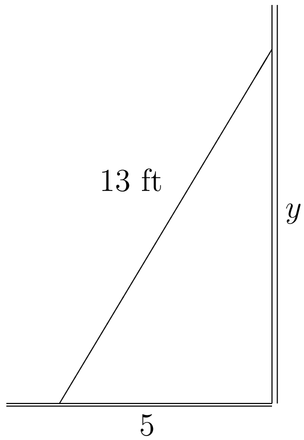
$$\begin{aligned}
y &= x^{\tan x} \\
\ln y &= \ln x^{\tan x} \\
\ln y &= (\tan x)(\ln x) \\
\frac{1}{y}\cdot y' &= (\tan x)'(\ln x) + (\tan x)(\ln x)' \\
\frac{1}{y}\cdot y' &= (\sec^2 x)(\ln x) + (\tan x)(1/x) \\
y' &= y\left((\sec^2 x)(\ln x) + \frac{\tan x}{x}\right) \\
y' &= x^{\tan x}\left((\sec^2 x)(\ln x) + \frac{\tan x}{x}\right)
\end{aligned}$$

13. A ladder 13 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 ft from the wall?

Solution:



- Facts: $\dfrac{dx}{dt} = 2\text{ft/s}$
- Find: $\dfrac{dy}{dt}$ when $x = 5\text{ft}$.
- Formula: $x^2 + y^2 = (13)^2$
- Derivative:
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
- Substitute $x = 5$ and $y = ?$.
  What is y when $x = 5$?



This is a Pythagorean Triplet, $5, 12, 13$.

$$5^2 + y^2 = (13)^2$$

By using the Pythagorean Theorem, we see that $y = 12$.

Now we can substitute $x = 5$, $y = 12$, $dx/dt = 2$ into

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0.$$

$$2(5)(2) + 2(12)\left.\frac{dy}{dt}\right|_{x=5} = 0.$$

We finally get

$$\left.\frac{dy}{dt}\right|_{x=5} = -\frac{5}{6} \text{ ft.}$$

Formulas

- Linearization of f at a: $\quad L(x) = f(a) + f'(a)(x - a)$
- Differential: $dy = \left(\frac{dy}{dx}\right)\, dx$

14. Find the linearization $L(x)$ of the function at a.

$$f(x) = \ln x, \quad a = 1$$

SOLUTION

$$\begin{aligned}
f(x) &= \ln x \\
f(a) &= f(1) = \ln 1 = 0 \\
f'(x) &= 1/x \\
f'(a) &= f'(1) = 1/1 = 1 \\
L(x) &= f(a) + f'(a)(x-a) \\
L(x) &= 0 + 1 \cdot (x-1) \\
L(x) &= x - 1
\end{aligned}$$

15. (a) Find the differential dy and (b) evaluate dy for the given values of x and dx.

$$y = 1/(x+1), x = 1, \ dx = -0.01$$

SOLUTION (a)

$$\begin{aligned}
y &= (x+1)^{-1} \\
dy/dx &= (-1)(x+1)^{-2} = -1/(x+1)^2
\end{aligned}$$

$$dy = \left(\frac{dy}{dx}\right) \ dx$$

$$dy = \left(-\frac{1}{(x+1)^2}\right) \ dx$$

(b) At $x = 1, \ dx = -0.01$,

$$dy = \left(-\frac{1}{(x+1)^2}\right)\ dx = \left(-\frac{1}{(1+1)^2}\right)\cdot(-0.01)$$

$$= \left(-\frac{1}{4}\right)(-0.01) = (-0.25)(-0.01) = 0.0025$$

16. *Hyperbolic Functions*:

Formulas

$\frac{d}{dx}(\sinh x) = \cosh x$ $\qquad$ $\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\text{sech } x) = -\text{msech } x \tanh x$ $\qquad$ $\frac{d}{dx}(\text{csch } x) = -\text{csch } x \coth x$

$\frac{d}{dx}(\tanh x) = \text{sech}^2 x$ $\qquad$ $\frac{d}{dx}(\coth x) = -\text{csch}^2 x$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}(\coth^{-1} x) = -\frac{1}{1-x^2}$$

$$\frac{d}{dx}(\text{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}} \qquad \frac{d}{dx}(\text{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

Calculate y'.

(a) $y = \ln(\cosh x)$

SOLUTION

$$y' = \left(\frac{1}{\cosh x}\right)\cdot \sinh x = \tanh x$$

(b) $y = \tanh^{-1}(\sqrt{x})$

SOLUTION

$$y' = \left(\frac{1}{1-(\sqrt{x})^2}\right)\cdot \frac{d}{dx}(x^{1/2})$$

$$= \left(\frac{1}{1-x}\right) \cdot \frac{1}{2}x^{-1/2}$$

$$= \left(\frac{1}{1-x}\right) \cdot \frac{1}{2\sqrt{x}}$$