

1.8 HIGHER-ORDER DERIVATIVES*Second and Higher Order Derivatives*

The first derivative of y with respect to x is denoted y' or $\frac{dy}{dx}$. The second derivative of y with respect to x is denoted y'' or $\frac{d^2y}{dx^2}$. The second derivative is an example of a higher – order derivative. We can continue to take derivatives (as long as they exist) using the following notation:

| | | | | |
|--|-----------|--------------|----------------------|--------------------------|
| <i>First derivative</i> | y' | $f'(x)$ | $\frac{dy}{dx}$ | $\frac{d}{dx}[f(x)]$ |
| <i>Second derivative</i> | y'' | $f''(x)$ | $\frac{d^2y}{dx^2}$ | $\frac{d^2}{dx^2}[f(x)]$ |
| <i>Third derivative</i> | y''' | $f'''(x)$ | $\frac{d^3y}{dx^3}$ | $\frac{d^3}{dx^3}[f(x)]$ |
| <i>Fourth derivative</i> | $y^{(4)}$ | $f^{(4)}(x)$ | $\frac{d^4y}{dx^4}$ | $\frac{d^4}{dx^4}[f(x)]$ |
| \vdots | \vdots | \vdots | \vdots | \vdots |
| <i>n^{th} derivative</i> | $y^{(n)}$ | $f^{(n)}(x)$ | $\frac{d^n y}{dx^n}$ | $\frac{d^n}{dx^n}[f(x)]$ |

Example: Find $\frac{d^4}{dx^4}[-5x^8 + 2x^6 - 9x^3 + 32x - 1]$.

Example: Let $f(x) = \frac{x}{x-1}$. Find $f''(x)$.

Example: If $f^{(4)}(x) = 2\sqrt{x}$, find $f^{(5)}(x)$.

Position – Velocity – Acceleration

We have already seen (courtesy of Wile E. Coyote) that the instantaneous rate of change in an object's position is velocity.

In other words, if $s(t)$ is a function of an object's position, then _____ is the function of the object's velocity.

The next question to arise ... What would the rate of change in velocity be?

Relationship of Position – Velocity – Acceleration

$$s(t) = \text{position at time } t$$

$$s'(t) = v(t) = \text{velocity at time } t$$

$$s''(t) = v'(t) = a(t) = \text{acceleration at time } t$$

Example: Bugs Bunny has been captured by Yosemite Sam and forced to “walk the plank”. Instead of waiting for Yosemite Sam to finish cutting the board from underneath him, Bugs finally decides just to jump. Bugs' position, s , is given by

$$s(t) = -16t^2 + 16t + 320$$

where s is measured in feet and t is measured in seconds.

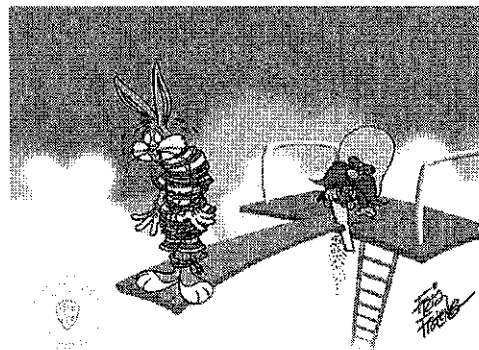
a) What is Bugs' maximum height?

b) When will Bugs hit the ground?

c) Find the velocity function for Bugs Bunny's freefall.

d) What is Bugs' speed at impact? (What are the units of the value?)

e) Find Bugs' acceleration as a function of time. (What are the units of this value?)



Higher Order Derivatives

For each problem, find the indicated derivative with respect to x .

1) $y = -x^2$ Find $\frac{d^2y}{dx^2}$

2) $f(x) = 4x^3$ Find f''

3) $y = -4x$ Find $\frac{d^3y}{dx^3}$

4) $f(x) = 5x^4$ Find f'''

5) $y = -5x^4$ Find $\frac{d^4y}{dx^4}$

6) $y = 3x^5 - 2x$ Find $\frac{d^3y}{dx^3}$

7) $y = -2x^3 - 4x^{-3}$ Find $\frac{d^3y}{dx^3}$

8) $y = -x^2 + 2\sqrt[5]{x^2}$ Find $\frac{d^3y}{dx^3}$

Critical thinking questions. Find the indicated derivatives with respect to x .

9) $y = 99x^{99}$ Find $\frac{d^{100}y}{dx^{100}}$

10) $f(x) = x^{99}$ Find $f^{(99)}$

Higher Order Derivatives

For each problem, find the indicated derivative with respect to x .

1) $y = -x^2$ Find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = -2$$

2) $f(x) = 4x^3$ Find f''

$$f''(x) = 24x$$

3) $y = -4x$ Find $\frac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = 0$$

4) $f(x) = 5x^4$ Find f'''

$$f'''(x) = 120x$$

5) $y = -5x^4$ Find $\frac{d^4y}{dx^4}$

$$\frac{d^4y}{dx^4} = -120$$

6) $y = 3x^5 - 2x$ Find $\frac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = 180x^2$$

7) $y = -2x^3 - 4x^{-3}$ Find $\frac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = -12 + \frac{240}{x^6}$$

8) $y = -x^2 + 2\sqrt[5]{x^2}$ Find $\frac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = \frac{96}{125x^5}$$

Critical thinking questions. Find the indicated derivatives with respect to x .

9) $y = 99x^{99}$ Find $\frac{d^{100}y}{dx^{100}}$

The 99th derivative is a constant, so 100th derivative is 0.

10) $f(x) = x^{99}$ Find $f^{(99)}$

99! (Made easy by factorial notation)