

### 1.17 Have You Reached the Limit?

304. Estimate the value of  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x + 1} - x)$  by graphing or by making a table of values.

305. Estimate the value of  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - \sqrt{x^2 - x})$  by graphing or by making a table of values.

306. Consider the function  $f(x) = \begin{cases} x^2 - 1 & -1 \leq x < 0 \\ 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x + 4 & 1 < x < 2 \\ 0 & 2 < x < 3. \end{cases}$

- a) Graph this function.
- b) Does  $f(-1)$  exist?
- c) Does  $\lim_{x \rightarrow -1^+} f(x)$  exist?
- d) Does  $\lim_{x \rightarrow -1^+} f(x) = f(-1)$ ?
- e) Is  $f$  continuous at  $x = -1$ ?
- f) Does  $f(1)$  exist?
- g) Does  $\lim_{x \rightarrow 1^+} f(x)$  exist?
- h) Does  $\lim_{x \rightarrow 1^+} f(x) = f(1)$ ?
- i) Is  $f$  continuous at  $x = 1$ ?
- j) Is  $f$  defined at  $x = 2$ ?
- k) Is  $f$  continuous at  $x = 2$ ?
- l) At what values of  $x$  is  $f$  continuous?
- m) What value should be assigned to  $f(2)$  to make the function continuous at  $x = 2$ ?
- n) To what new value of  $f(1)$  be changed to remove the discontinuity?

307. Is  $F(x) = \frac{|x^2 - 4|x}{x + 2}$  continuous everywhere? Why or why not?

308. Is  $F(x) = \frac{|x^2 + 4x|(x + 2)}{x + 4}$  continuous everywhere? Why or why not?

FIND THE CONSTANTS  $a$  AND  $b$  SUCH THAT THE FUNCTION IS CONTINUOUS EVERYWHERE.

309.  $f(x) = \begin{cases} x^3 & x \leq 2 \\ ax^2 & x > 2 \end{cases}$

310.  $g(x) = \begin{cases} \frac{4 \sin x}{x} & x < 0 \\ a - 2x & x \geq 0 \end{cases}$

311.  $f(x) = \begin{cases} 2 & x \leq -1 \\ ax + b & -1 < x < 3 \\ -2 & x \geq 3 \end{cases}$

312.  $g(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & x \neq a \\ 8 & x = a \end{cases}$

### 1.9 Limits Determined by Tables

USING YOUR CALCULATOR, FILL IN EACH OF THE FOLLOWING TABLES TO FIVE DECIMAL PLACES. USING THE INFORMATION FROM THE TABLE, DETERMINE EACH LIMIT. (FOR THE TRIGONOMETRIC FUNCTIONS, YOUR CALCULATOR MUST BE IN *radian* MODE.)

164.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$

$x$	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{\sqrt{x+3} - \sqrt{3}}{x}$						

165.  $\lim_{x \rightarrow -3} \frac{\sqrt{1-x} - 2}{x+3}$

$x$	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9
$\frac{\sqrt{1-x} - 2}{x+3}$						

166.  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

$x$	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{\sin x}{x}$						

167.  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

$x$	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{1 - \cos x}{x}$						

168.  $\lim_{x \rightarrow 0} (1+x)^{1/x}$

$x$	-0.1	-0.01	-0.001	0.001	0.01	0.1
$(1+x)^{1/x}$						

169.  $\lim_{x \rightarrow 1} x^{1/(1-x)}$

$x$	0.9	0.99	0.999	1.001	1.01	1.1
$x^{1/(1-x)}$						

Science is built up with facts, as a house is with stones. But a collection of facts is no more a science than a heap of stones is a house. —*Henri Poincaré*

### 1.8 Limits Determined by Graphs

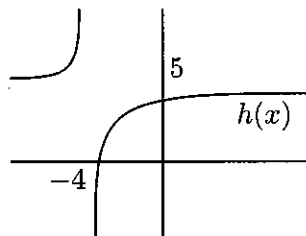
REFER TO THE GRAPH OF  $h(x)$  TO EVALUATE THE FOLLOWING LIMITS.

142.  $\lim_{x \rightarrow -4^+} h(x)$

143.  $\lim_{x \rightarrow -4^-} h(x)$

144.  $\lim_{x \rightarrow \infty} h(x)$

145.  $\lim_{x \rightarrow -\infty} h(x)$



REFER TO THE GRAPH OF  $g(x)$  TO EVALUATE THE FOLLOWING LIMITS.

146.  $\lim_{x \rightarrow a^+} g(x)$

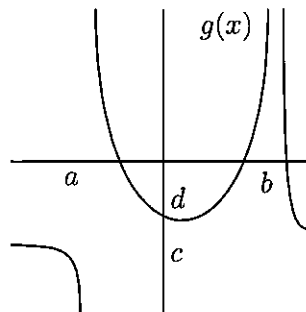
151.  $\lim_{x \rightarrow b^-} g(x)$

147.  $\lim_{x \rightarrow a^-} g(x)$

148.  $\lim_{x \rightarrow 0} g(x)$

149.  $\lim_{x \rightarrow \infty} g(x)$

150.  $\lim_{x \rightarrow b^+} g(x)$



REFER TO THE GRAPH OF  $f(x)$  TO DETERMINE WHICH STATEMENTS ARE TRUE AND WHICH ARE FALSE. IF A STATEMENT IS FALSE, EXPLAIN WHY.

152.  $\lim_{x \rightarrow -1^+} f(x) = 1$

159.  $\lim_{x \rightarrow 1} f(x) = 1$

153.  $\lim_{x \rightarrow 0^-} f(x) = 0$

160.  $\lim_{x \rightarrow 1} f(x) = 0$

154.  $\lim_{x \rightarrow 0^-} f(x) = 1$

161.  $\lim_{x \rightarrow 2^-} f(x) = 2$

155.  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$

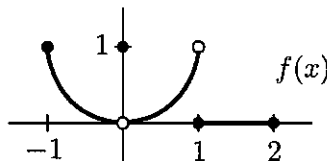
162.  $\lim_{x \rightarrow -1^-} f(x)$  does not exist

156.  $\lim_{x \rightarrow 0} f(x)$  exists

163.  $\lim_{x \rightarrow 2^+} f(x) = 0$

157.  $\lim_{x \rightarrow 0} f(x) = 0$

158.  $\lim_{x \rightarrow 0} f(x) = 1$



## 1.15 Solving Equations

SOLVE EACH OF THE FOLLOWING EQUATIONS.

245.  $1 - \frac{8}{k^3} = 0$

246.  $4p^3 - 4p = 0$

247.  $x^3 - 2x^2 - 3x = 0$

248.  $3x^2 - 10x - 8 = 0$

249.  $|4x^3 - 3| = 0$

250.  $|w^2 - 6w| = 9$

251.  $\frac{3(x-4) - (3x-2)}{(x-4)^2} = 0$

252.  $\frac{2x-3}{2(x^2-3x)} = 0$

253.  $2 \ln x = 9$

254.  $e^{5x} = 7$

255.  $\ln(2x-1) = 0$

256.  $e^{3x+7} = 12$

257.  $\ln \sqrt[4]{x+1} = \frac{1}{2}$

258.  $2^{3x-1} = \frac{1}{2}$

259.  $\log_8(x-5) = \frac{2}{3}$

260.  $\log \sqrt{z} = \log(z-6)$

261.  $2 \ln(p+3) - \ln(p+1) = 3 \ln 2$

262.  $3^{x^2} = 7$

263.  $\log_3(3x) = \log_3 x + \log_3(4-x)$

FIND ALL REAL ZEROS OF THE FOLLOWING FUNCTIONS.

264.  $y = x^2 - 4$

265.  $y = -2x^4 + 5$

266.  $y = x^3 - 3$

267.  $y = x^3 - 9x$

268.  $y = x^4 + 2x^2$

269.  $y = x^3 - 4x^2 - 5x$

270.  $y = x^3 - 5x^2 - x + 5$

271.  $y = x^3 + 3x^2 - 4x - 12$

272.  $y = \frac{x-2}{x}$

273.  $y = \frac{-1}{(x-1)^2}$

274.  $y = \frac{1+x}{1-x}$

275.  $y = \frac{x^3}{1+x^2}$

276.  $y = \frac{x^2 - 2x}{x^2 - 16}$

277.  $y = \frac{x^2 - 4x + 3}{x-4}$

278.  $y = \frac{x^3 + 3x^2}{x^4 - 4x^2}$

279.  $y = \frac{x^5 - 25x^3}{x^4 + 2x^3}$

280.  $y = x^2 + \frac{1}{x}$

281.  $y = e^{3x-1} \sqrt{x}$

282.  $y = x \log_3(5x-2)$

283.  $y = e^{3x/(2x-1)} \sqrt[3]{x-7}$

284.  $y = \ln(8x^2 - 4)$

285.  $y = e^{5x/(3x-2)} \ln e^x$

DETERMINE WHETHER THE FUNCTIONS IN THE PROBLEMS LISTED ARE EVEN, ODD, OR NEITHER.

286. problem 264

288. problem 272

290. problem 275

287. problem 268

289. problem 274

291. problem 280

The chief aim of all investigations of the external world should be to discover the rational order and harmony which has been imposed on it by God and which He revealed to us in the language of mathematics. — *Johannes Kepler*