

**Area, Volume and
Perpendicular Cross Sections**

AP Free Response Questions

Years 2002 to 2014

Hopkins

2002 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A**

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$. Show the analysis that leads to your answers.

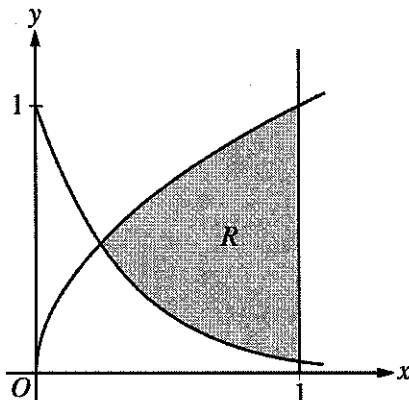
2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A

Time—45 minutes

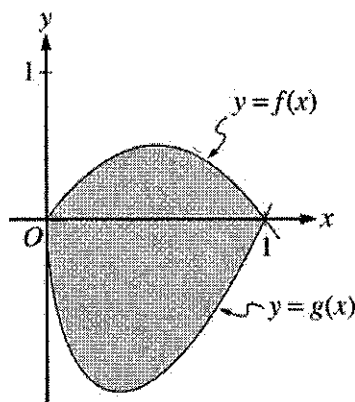
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
 - (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
 - (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is 5 times the length of its base in region R . Find the volume of this solid.
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2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \leq x \leq 1$. The graphs of f and g are shown in the figure above.
- Find the area of the shaded region enclosed by the graphs of f and g .
 - Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.
 - Let h be the function given by $h(x) = kx(1 - x)$ for $0 \leq x \leq 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x -axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k .

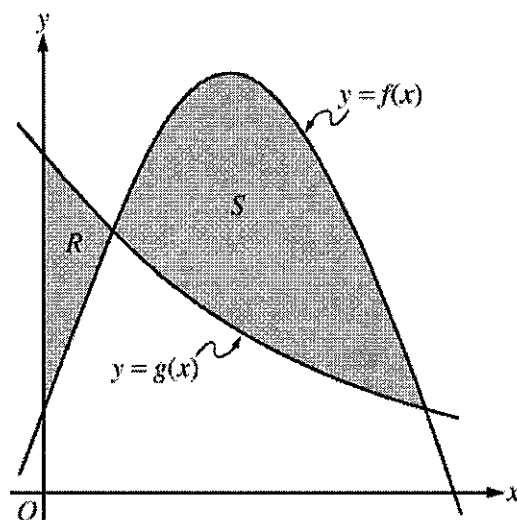
2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A**

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.
- (a) Find the area of R .
- (b) Find the area of S .
- (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

WRITE ALL WORK IN THE TEST BOOKLET.

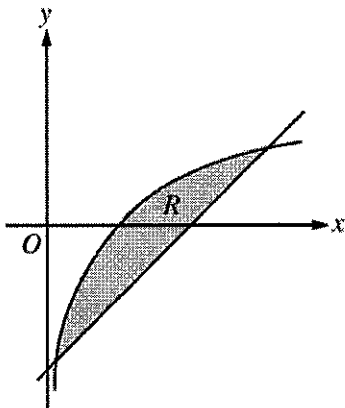
2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A**

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.
- Find the area of R .
 - Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.
 - Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y -axis.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3**

A graphing calculator is required for some problems or parts of problems.

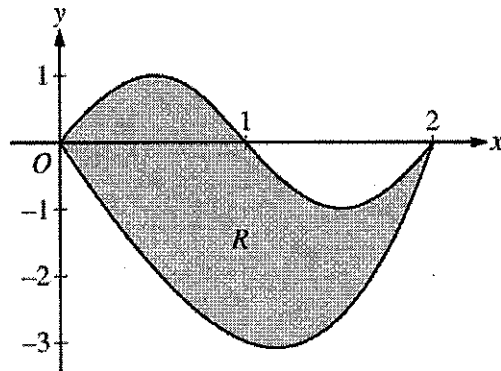
1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.
- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is rotated about the x -axis.
- (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3**

A graphing calculator is required for some problems or parts of problems.



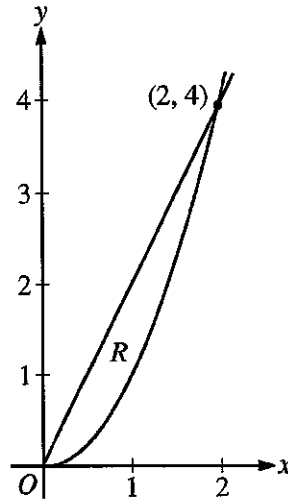
- Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.
 - Find the area of R .
 - The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.
 - The region R models the surface of a small pond. At all points in R at a distance x from the y -axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2009 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.
- (a) Find the area of R .
- (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
- (c) Another solid has the same base R . For this solid, the cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

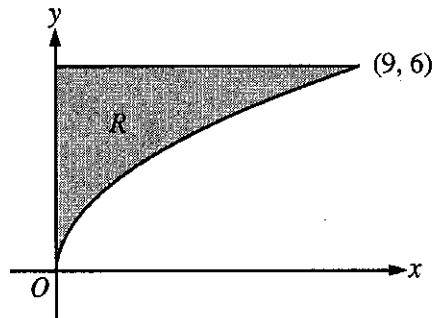
2010 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



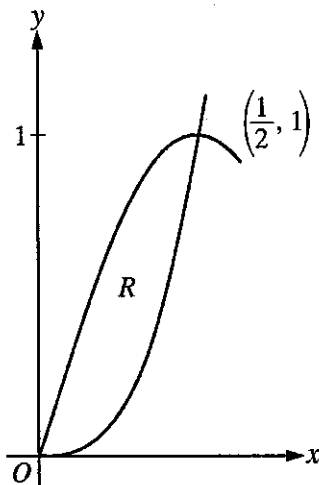
4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y -axis, as shown in the figure above.
- (a) Find the area of R .
 - (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
 - (c) Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2011 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

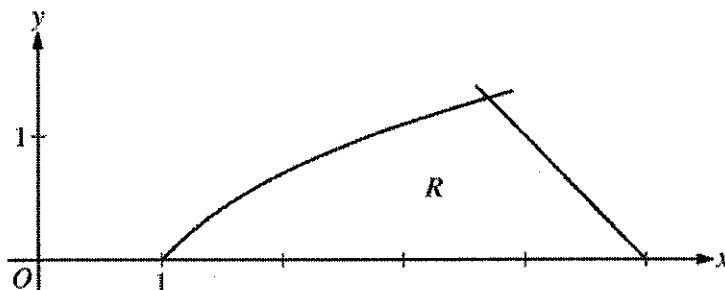
No calculator is allowed for these problems.



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.
- Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
 - Find the area of R .
 - Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

WRITE ALL WORK IN THE EXAM BOOKLET.

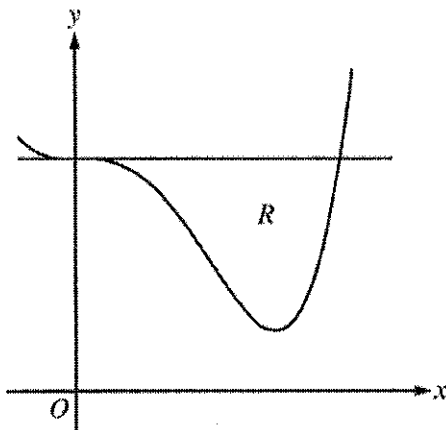
2012 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



2. Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.
- Find the area of R .
 - Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
 - The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .
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END OF PART A OF SECTION II

2014 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.
- (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
 - (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
 - (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .
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END OF PART A OF SECTION II