

## AP Calculus BC Advanced Integration Techniques Practice Test

Directions:

- Copy each problem onto your own paper and use a proper heading.
- Justify all answers, responses, and steps with appropriate mathematics.

Determine each antiderivative.

$$(1) \int \frac{x+4}{x^2+x-2} dx$$

$$(2) \int (x \cdot \sin(3x)) dx$$

$$(3) \int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$(4) \int x^2 e^{-x} dx$$

$$(5) \int \frac{2x-19}{x^2+x-6} dx$$

$$(6) \int \frac{\sqrt{x^2-3}}{x} dx$$

$$(7) \int \frac{dx}{x^2\sqrt{x^2+1}}$$

$$(8) \int \ln(3x+2) dx$$

$$(9) \int \frac{x+1}{(x+2)^2} dx$$

$$(10) \int \frac{dx}{(x^2+25)^2}$$

$$(11) \int (e^x \cdot \cos(x)) dx$$

$$(12) \int \frac{(\ln(x))^2}{x} dx$$

$$(13) \int \frac{x^3-4x^2-3x+3}{x^2-3x} dx$$

$$(14) \int x \sqrt{4-x^2} dx$$

$$(15) \int \frac{\sec^2(\theta)}{\tan(\theta)(\tan(\theta)-1)} d\theta$$

$$(16) \int x \sqrt{5x-2} dx$$

$$(17) \int \sqrt{25-9x^2} dx$$

$$(18) \int \arctan(2x) dx$$

Determine if each improper integral converges, or diverges. If the improper integral converges, show what the integral converges to. If the improper integral diverges, write diverges.

$$(19) \int_1^{\infty} \frac{dx}{x^{2.001}}$$

$$(20) \int_{-\infty}^0 e^{5x} dx$$

$$(21) \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

$$(22) \int_3^5 \frac{dx}{(x-3)^4}$$

$$(23) \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$(24) \int_1^7 \frac{x+2}{x-3} dx$$

## ANSWERS to AP Calculus BC Advanced Integration Techniques Practice Test

$$(1) \quad C + \ln \sqrt[3]{\frac{|x-1|^5}{(x+2)^2}}$$

$$(2) \quad \frac{1}{9}(\sin(3x) - 3x\cos(3x)) + C$$

$$(3) \quad \frac{-\sqrt{4-x^2}}{4x} + C$$

$$(4) \quad C - e^{-x}(x^2 + 2x + 2)$$

$$(5) \quad C + \ln \frac{|x+3|^5}{|x-2|^3}$$

$$(6) \quad \sqrt{x^2-3} - \sqrt{3} \cdot \arccos\left(\frac{\sqrt{3}}{x}\right) + C$$

$$(7) \quad C - \frac{\sqrt{x^2+1}}{x}$$

$$(8) \quad C + \ln \left( \frac{(3x+2)^{\frac{3x+2}{3}}}{e^x} \right)$$

$$(9) \quad \frac{1+(x+2)\ln|x+2|}{x+2} + C$$

$$(10) \quad \frac{1}{250} \left( \arctan\left(\frac{x}{5}\right) + \frac{5x}{x^2+25} \right) + C$$

$$(11) \quad \frac{e^x}{2} (\sin(x) + \cos(x)) + C$$

$$(12) \quad \frac{(\ln(x))^3}{3} + C$$

$$(13) \quad x^2 - x - \ln|x(x-3)^5| + C$$

$$(14) \quad \frac{-(4-x^2)\sqrt{4-x^2}}{3} + C$$

$$(15) \quad \ln|1 - \cot(\theta)| + C$$

$$(16) \quad \frac{2(5x-2)\sqrt{5x-2}}{375} (15x+4) + C$$

$$(17) \quad \frac{1}{6} \left( 25\arcsin\left(\frac{3x}{5}\right) + 3x\sqrt{25-9x^2} \right) + C$$

$$(18) \quad x \cdot \arctan(2x) - 0.25\ln(4x^2+1) + C$$

$$(19) \quad \text{converges to } 1000/1001$$

$$(20) \quad \text{converges to } 1/5$$

$$(21) \quad \text{converges to } \pi$$

$$(22) \quad \text{diverges}$$

$$(23) \quad \text{converges to } (\pi)/2$$

$$(24) \quad \text{converges to } 6+5\ln(2)$$