AB CALCULUS

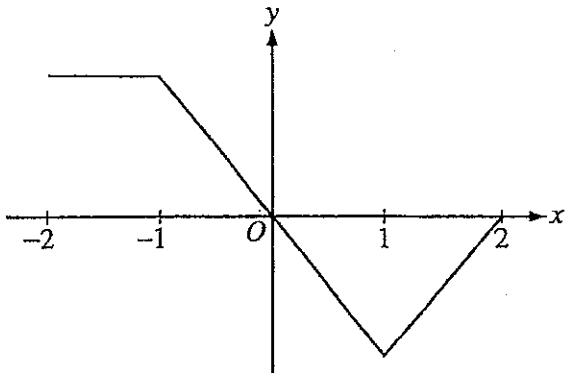
APPLICATIONS OF THE DERIVATIVE

MULTIPLE CHOICE QUESTIONS

1969 – 2003

HOPKINS

2003 AB



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

(A) f is decreasing for $-1 \le x \le 1$.

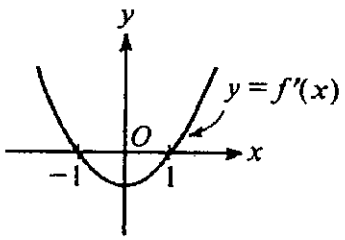
(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$.

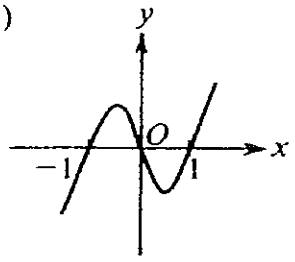
(E) f is not differentiable at $x = -1$ and $x = 1$.

1985

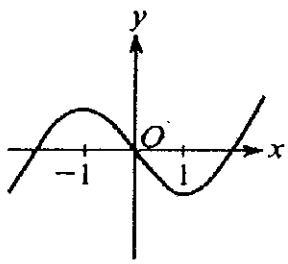


33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

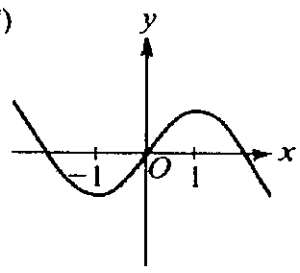
(A)



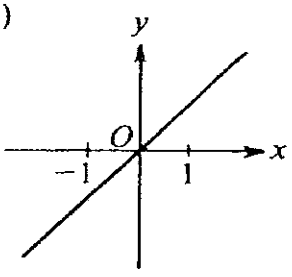
(B)



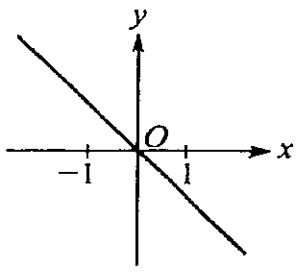
(C)



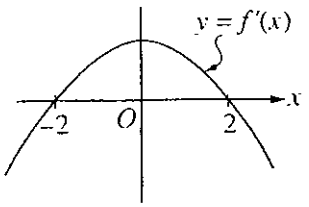
(D)



(E)

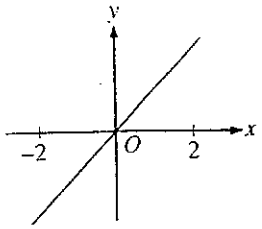


1997
AB

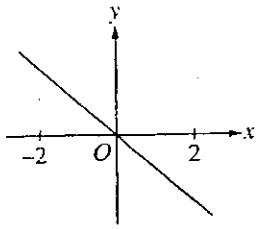


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

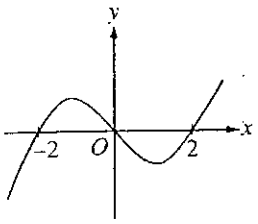
(A)



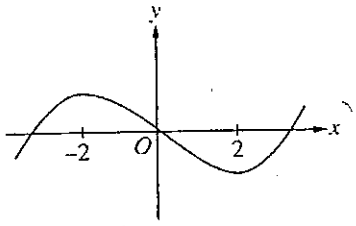
(B)



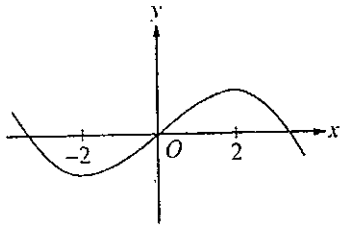
(C)



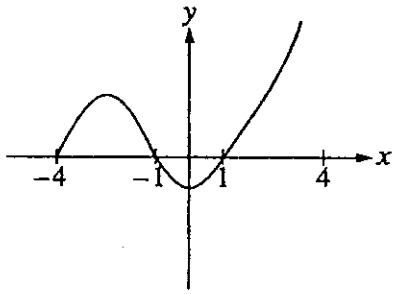
(D)



(E)

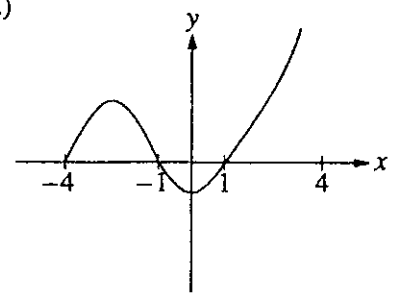


1997
AB
1993
AB

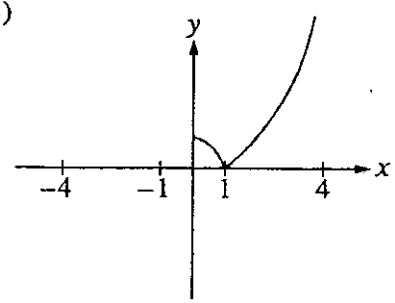


40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

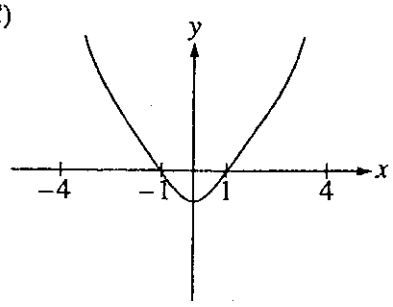
(A)



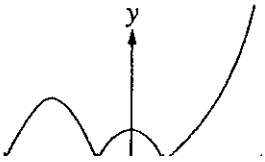
(B)



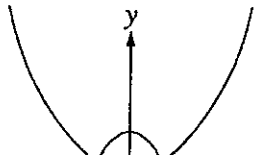
(C)



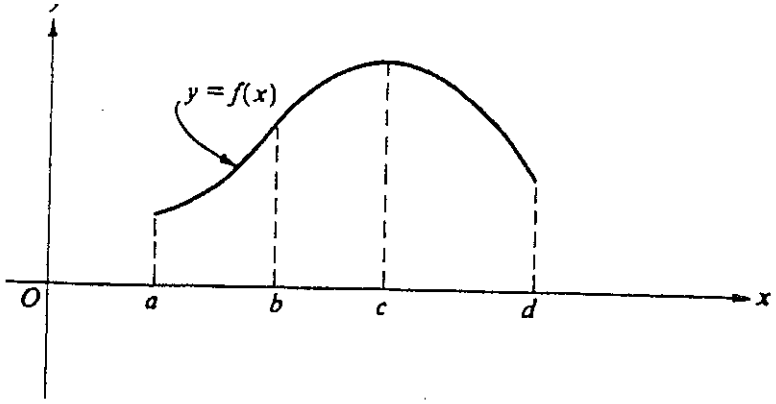
(D)



(E)



1956 AB

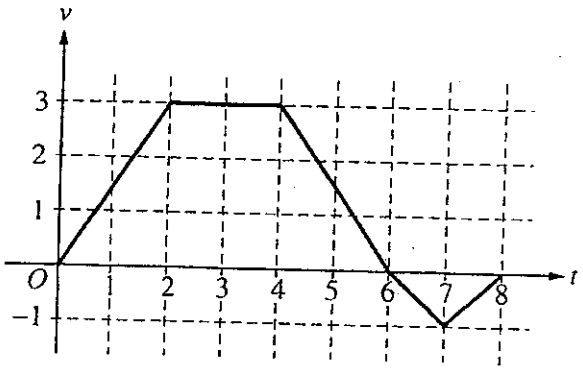


8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

1997 AB



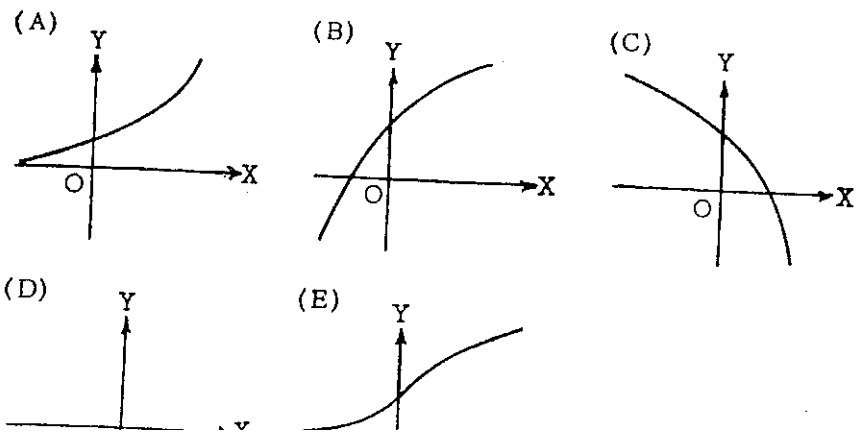
A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

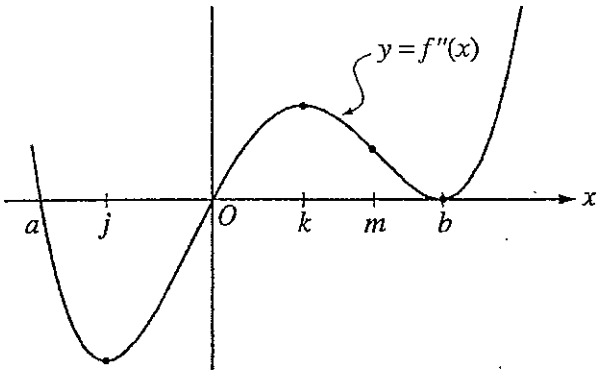
(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

1969 AB



2003 AB #21



21. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k

1998 AB

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

2003 AB

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

1969 AB

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.

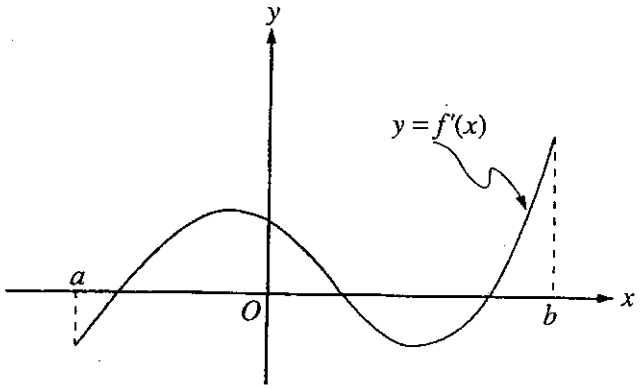
(B) f is decreasing.

(C) f is discontinuous.

(D) f has a relative minimum.

(E) f has a relative maximum.

1997 BC



12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b)?

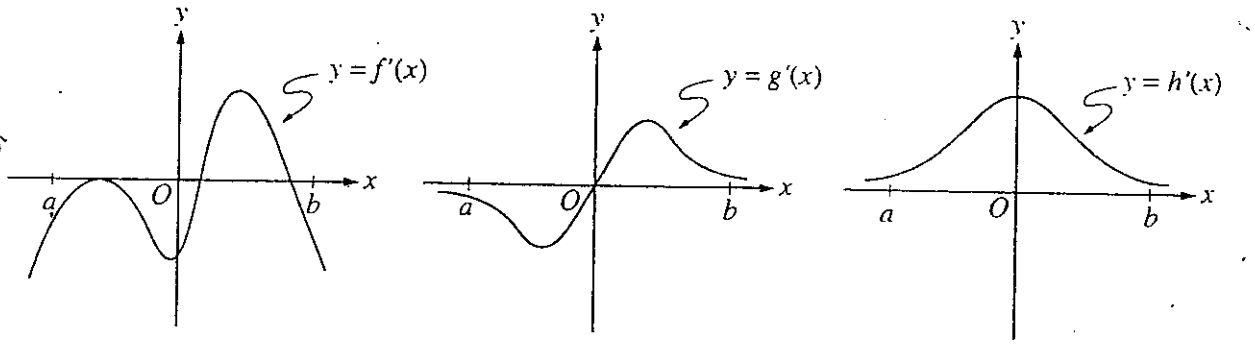
(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right) g(x)$, which of the following is true?

1998 AB rel. min/max

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

1998 AB rel. max



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

1998 AB rel. max

7. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of the

16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

2. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

(A) -1 only (B) 0 only (C) 1 only (D) -1 and 1 only (E) -1, 0 and 1

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

15. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

44. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

3. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) -1 (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

10. The *derivative* of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$

(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

30. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x = -1$ and $x = 3$.
(B) $f'(-1) = 0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x = 3$.
(E) The graph of f intersects both axes.

39. If $f(x)=\dfrac{\ln x}{x}$, for all $x>0$, which of the following is true?

1985

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

---

3. If $f(x)=x+\dfrac{1}{x}$, then the set of values for which f increases is

1973 BC

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $[-1,1]$ (C) $(-\infty,\infty)$
(D) $(0,\infty)$ (E) $(-\infty,0)\cup(0,\infty)$

---

27. The function f given by $f(x)=x^3+12x-24$ is

1993 AB

(A) increasing for $x<-2$, decreasing for $-2<x<2$, increasing for $x>2$
(B) decreasing for $x<0$, increasing for $x>0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x<-2$, increasing for $-2<x<2$, decreasing for $x>2$

1997 AB

13. Let f be a function defined for all real numbers x. If $f'(x)=\dfrac{|4-x^2|}{x-2}$, then f is decreasing on the interval

(A) $(-\infty,2)$ (B) $(-\infty,\infty)$ (C) $(-2,4)$ (D) $(-2,\infty)$ (E) $(2,\infty)$

---

22. What are all values of x for which the function f defined by $f(x)=(x^2-3)e^{-x}$ is increasing?

1997 AB

(A) There are no such values of x.
(B) $x<-1$ and $x>3$
(C) $-3<x<1$
(D) $-1<x<3$
(E) All values of x

---

87. The function f has first derivative given by $f'(x)=\dfrac{\sqrt{x}}{1+x+x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

2008 AB P01

(A) 1.008 (B) 0.473 (C) 0 (D) -0.278 (E) The graph of f has no inflection point.

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

1997 AB concavity

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

2003 AB Concavity

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

---

77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

1997 AB

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

---

21. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

1998 AB POI

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

'98 AB POI

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

---

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

1998 AB POI

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

---

17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

1969 AB

(A) (0,0) only (B) (3,162) only (C) (4,256) only

(D) (0,0) and (3,162) (E)

1985 PVA

14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

1985 PVA

11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

---

26. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

1993 AB PVA

(A) –5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

---

87. At time $t \ge 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is –2. For what value t will the velocity of the particle be zero?

1997 AB PVA

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

---

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

1998 AB PVA

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

1998 AB PVA

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

1973 CONCAVITY

(A) $x > 0$

(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

(C) $-2 < x < 0$ or $x > 2$

(D) $x > \sqrt{2}$

(E) $-2 < x < 2$

---

4. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

1988 CONCAVITY

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

2008 AB

31. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

---

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \dfrac{\ln t}{t}$. At what value of t does v attain its maximum?

1989 AB

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

---

23. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

1993 AB C.P.

(A) One (B) Two (C) Three (D) Five (E) Nine

---

18. If $f(x) = \sin\left(\dfrac{x}{2}\right)$, then there exists a number c in the interval $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

1993 AB Mean Value Theorem

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{4}$ (C) $\dfrac{5\pi}{6}$ (D) π (E) $\dfrac{3\pi}{2}$

---

11. The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\dfrac{1}{2}\right)$ occurs where y is

1969 AB Fr. Opt.

(A) $\dfrac{1}{2}$ (B) 0 (C) $-\dfrac{1}{2}$ (D) -1 (E) none of the above

---

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \dfrac{4}{3}\pi r^3\right)$

1973 related rates

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

---

39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

1993 AB el. rates

(A) $\dfrac{1}{\pi}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{2}{\pi}$ (D) 1 (E) 2

circumference C, what is the rate of change of the area of [illegible] second?

1998 AB rel. rates

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\dfrac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

---

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

1998 AB rel. rates

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

---

31. The volume of a cone of radius r and height h is given by $V = \dfrac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\dfrac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

1985 rel. rates

(A) $\dfrac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

---

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

2005 AB rel. rates

(A) 0.04π m^2/sec

(B) 0.4π m^2/sec

(C) 4π m^2/sec

(D) 20π m^2/sec

(E) 100π m^2/sec

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

2005 AB PVA

(A) –2.016 (B) –0.677 (C) 1.633 (D) 1.814 (E) 2.978