

AP Calculus

Hopkins

Separation of Variables and Slope Fields

Years 2003 to 2012

2011 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

5pts. (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

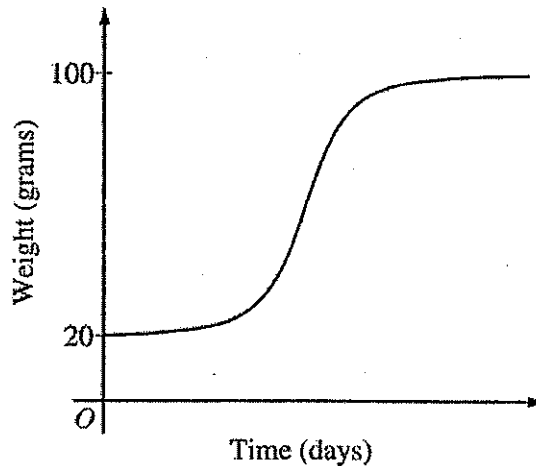
2012 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

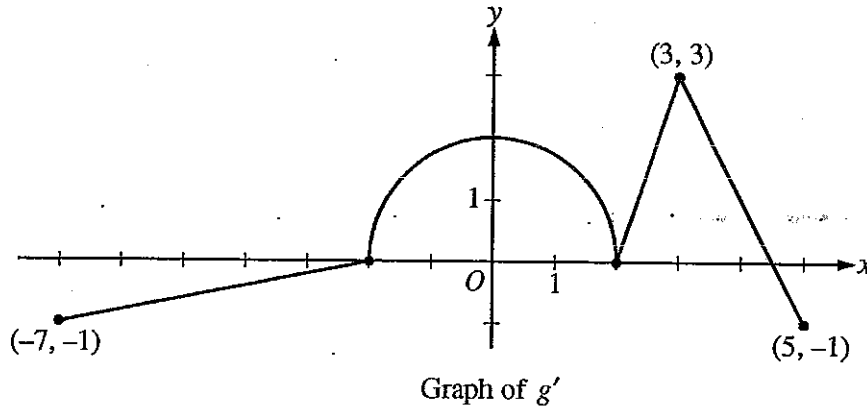
Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.
- (b) Find $\frac{d^2B}{dt^2}$ in terms of B . Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



- 5pts. (c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

2010 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g , consists of a semicircle and three line segments, as shown in the figure above.
- Find $g(3)$ and $g(-2)$.
 - Find the x -coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.
 - The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x -coordinate of each critical point of h , where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.
- Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.
 - Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.
- 5 pts. (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

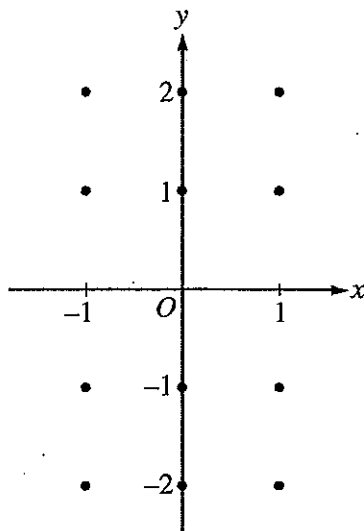
END OF EXAM

2010 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

3 pts. (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane for which $y \neq 0$. Describe all points in the xy -plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

5 pts. (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

6. Two particles move along the x -axis. For $0 \leq t \leq 6$, the position of particle P at time t is given by

$$p(t) = 2\cos\left(\frac{\pi}{4}t\right), \text{ while the position of particle } R \text{ at time } t \text{ is given by } r(t) = t^3 - 6t^2 + 9t + 3.$$

(a) For $0 \leq t \leq 6$, find all times t during which particle R is moving to the right.

(b) For $0 \leq t \leq 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \leq t \leq 3$.

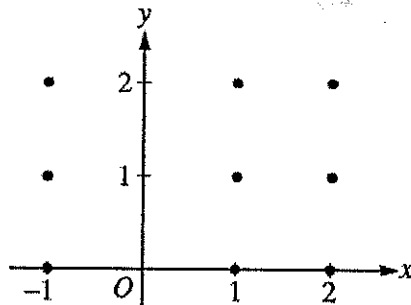
WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM

2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

- 2 pts. (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
 (Note: Use the axes provided in the exam booklet.)



- 6 pts. (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.
 (c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \rightarrow \infty} f(x)$.

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

- (a) Write an equation for the line tangent to the graph of f at $x = e^2$.
 (b) Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum, or neither for the function f . Justify your answer.
 (c) The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.
 (d) Find $\lim_{x \rightarrow 0^+} f(x)$.

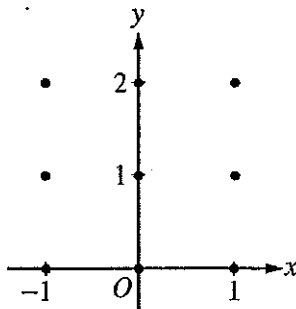
WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Describe the region in the xy -plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b , for which $y = mx + b$ is a solution to the differential equation.

6. Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all x , then the graph of g does not have a point of inflection.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

WRITE ALL WORK IN THE EXAM BOOKLET.

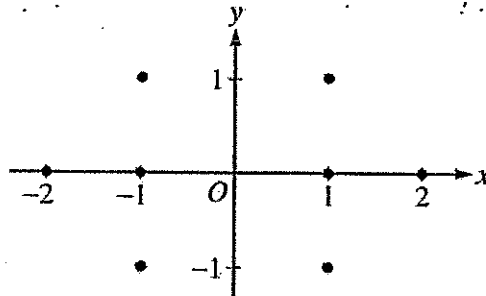
END OF EXAM

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

2 pts. (a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



7 pts. (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

- (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.
- (b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

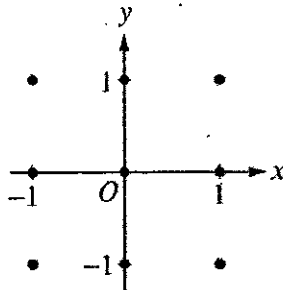
WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = (y - 1)^2 \cos(\pi x)$.

2 pts (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



6 (b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c .

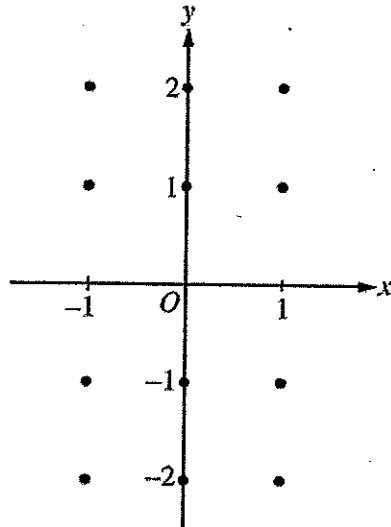
6 pts (c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

2 pt (Note: Use the axes provided in the pink test booklet.)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$.

Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

5 pt (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

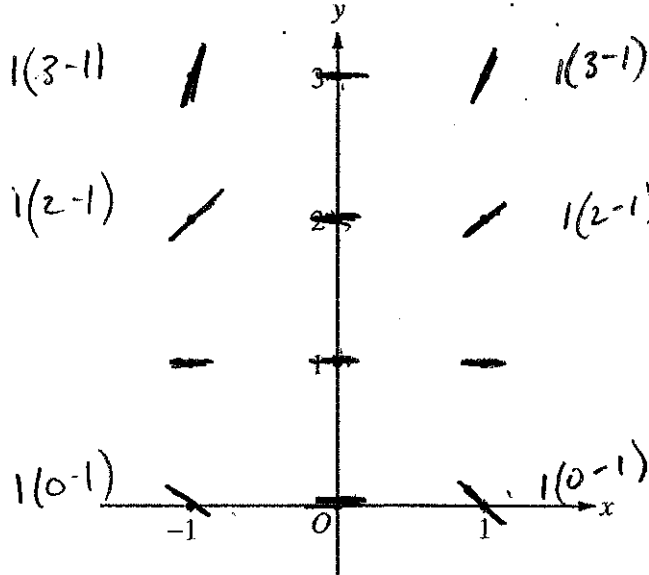
WRITE ALL WORK IN THE TEST BOOKLET.

END OF EXAM

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

2 pts (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (Note: Use the axes provided in the pink test booklet.)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are positive.

6 pts (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

$$C \quad \frac{dy}{dx} = x^2(y-1)$$

$$\int \frac{dy}{y-1} = \int x^2 dx$$

$$\ln|y-1| = \frac{x^3}{3} + C$$

$$|y-1| = e^{\frac{x^3}{3} + C} = e^{\frac{x^3}{3}} e^C$$

$$|y-1| = Ke^{\frac{x^3}{3}}$$

$$2 = Ke^0$$

$$2 = K$$

$$y-1 = 2e^{\frac{x^3}{3}}$$

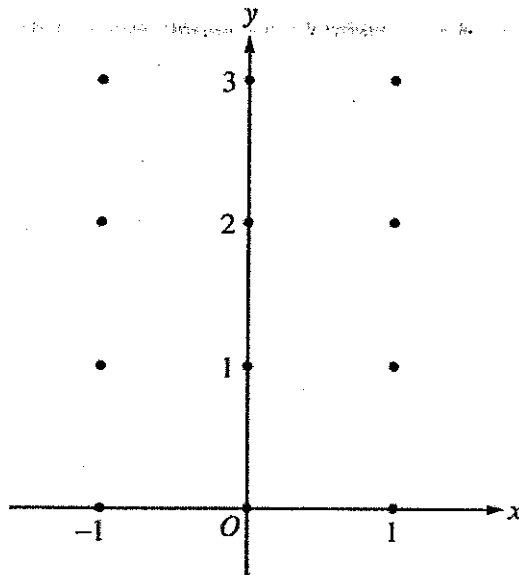
$$y = 2e^{\frac{x^3}{3}} + 1$$

END OF EXAMINATION

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = x^4(y - 2)$.

- 2 pts (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
 (Note: Use the axes provided in the test booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are negative.
- 6 pts (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.
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2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

(a) Find $f''(3)$.

6.7 (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

END OF EXAMINATION



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