## Math Worksheet ${ }_{1-}$ FUNCTION versus RELATION

## Relations

A relation is a set of inputs and outputs, often written as ordered pairs (input, output). We can also represent a relation as a mapping diagram or a graph. For example, the relation can be represented as:



## Mapping Diagram of Relation

Graph of Relation
$y$ is not a function of $x$ ( $x=0$ has multiple outputs)

## Functions

A function is a relation in which each input $x$ (domain) has only one output $y$ (range).



To check if a relation is a function, given a mapping diagram of the relation, use the following criterion:

1. If each input has only one line connected to it, then the outputs are a function of the inputs.
2. The Vertical Line Tests for Graphs

To determine whether $y$ is a function of $x$, given a graph of a relation, use the following criterion: if every vertical line you can draw goes through only 1 point, $y$ is a function of $x$. If you can draw a vertical line that goes through 2 points, $y$ is not a function of $x$. This is called the vertical line test.

In the following graphs:
$y$ is a function of $x$ (passes vertical line trest)

$y$ is not a function of $x$ (fails vertical line test)


## Function notation

There is a special notation, that is used to represent this situation:
if the function name is $\boldsymbol{f}$, and the input name is $\boldsymbol{x}$, then the unique corresponding output is called $\boldsymbol{f}(\boldsymbol{x})$ (which is read as " $\boldsymbol{f}$ of $\boldsymbol{x}$ ".)

We can also use letters: $\boldsymbol{g}(x), \boldsymbol{h}(x)$ or simply $\boldsymbol{y}$

Question: What does the function notation $g(7)$ represent?
Answer: the output from the function $g$ when the input is 7
Question: Suppose $f(x)=x+2$. What is $f(3)$ ?
Answer: $f(3)=3+2=5$ (simply substitute number 3 for the variable x )
Question: Suppose $f(x)=x+2$. What is $f(x+5)$ ?
Answer: $f(x+5)=(x+5)+2=x+7$

## Operations with functions

Given $f(x)=3 x+2$ and $g(x)=4-5 x$, find $(f+g)(x),(f-g)(x),(f \times g)(x)$, and $(f / g)(x)$.
$(f+g)(x)=f(x)+g(x)=[3 x+2]+[4-5 x]=3 x-5 x+2+4=-2 x+6$
$(f-g)(x)=f(x)-g(x)=[3 x+2]-[4-5 x]=3 x+5 x+2-4=8 x-2$
$(f \times g)(x)=[f(x)][g(x)]=(3 x+2)(4-5 x)=12 x+8-15 x^{2}-10 x=-15 x^{2}+2 x+8$

$$
(f / g)(x)=f(x) / g(x)=\frac{3 x+2}{4-5 x}
$$

## Exercises

State the domain and range of each relation. Then determine whether each relation is a function
1.

2.

3.

| $x$ | $y$ |
| :---: | :---: |
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |

4. 



Graph each relation or equation and determine the domain and range.
5. $\{(2,-3),(2,4),(2,-1)\}$

6. $\{(2,6),(6,2)\}$

7. $\{(-3,4),(-2,4),(-1,-1),(3,-1)\}$


Find each value if $f(x)=2 x-1$ and $g(x)=2-x 2$.
9. $f(0)$
10. $f(12)$
11. $g(4)$
12. $f(-2)$
13. $g(-1)$
14. $f(d)$

## Homework

State the domain and range of each relation. Then determine whether each relation is a function
1.

2.

3.

| $x$ | $y$ |
| :---: | :---: |
| -3 | 0 |
| -1 | -1 |
| 0 | 0 |
| 2 | -2 |
| 3 | 4 |

4. 

| $x$ | $y$ |
| :---: | :---: |
| -2 | -1 |
| -2 | 1 |
| -1 | 0 |
| 1 | 0 |
| 2 | 1 |

Graph each relation or equation and determine the domain and range.
5. $x=-1$

6. $y=2 x-1$


Find each value if $f(x)=-5 x+2$ and $g(x)=-2 x+3$.
7. $f(3)$
8. $f(-4)$
9. $g(-12)$
10. $f(-2)$
11. $g(-6)$
12. $f(m-2)$
13. Use the functions below to perform the following operations:
$\mathrm{f}(\mathrm{x})=2 \mathrm{x}$
$\mathrm{g}(\mathrm{x})=\mathrm{x}-2$
$h(x)=x^{2}$
$\mathrm{k}(\mathrm{x})=\mathrm{x} / 2$
$\mathrm{k}(\mathrm{x}) x \mathrm{f}(\mathrm{x})$
$\mathrm{g}(\mathrm{x})-\mathrm{h}(\mathrm{x})$
$\mathrm{f}(\mathrm{x})-\mathrm{k}(\mathrm{x})$
$\mathrm{h}(\mathrm{x})+\mathrm{k}(\mathrm{x})$
$\mathrm{f}(\mathrm{x}) \div \mathrm{k}(\mathrm{x})$
$\mathrm{g}(\mathrm{x}) \times \mathrm{h}(\mathrm{x})$

